# LIA in a Nut Shell: How can Trigonometry help to understand Lock-in Amplifier operation?

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**Abstract**

Taking into account the actual need and importance of introducing fundamental concepts related to the detection of small and noisy electrical signals in the curriculum of physics and engineering studies, in this work a simple trigonometric analysis often encountered in mathematics class room's exercises will be used to show a simple manner to familiarize students at basic undergraduate college and university level with the basic principles of the Lock-in Amplifier, an instrument widely used in research laboratories for the measurement of AC signals modulated at a given frequency in an environment of very low signal to noise ratio.

**Keywords:** Electronic measurement, electrical noise, lock-in amplifier, trigonometry.

**Resumen**

Teniendo en cuenta la necesidad y la importancia de introducir en el currículo de estudio de física e ingeniería los conceptos fundamentales relacionados con la detección de señales eléctricas pequeñas y ruidosas, en este trabajo utilizaremos un análisis trigonométrico sencillo, muchas veces encontrado en ejercicios de aula de matemática, para mostrar una manera relativamente simple de familiarizar a los estudiantes de pregrado con los principios básicos del Amplificador Lock-in, un instrumento ampliamente utilizado en laboratorios de investigación para medir señales de corriente alterna a determinada frecuencia, en un ambiente de muy baja relación señal-ruido.

**Palabras claves:** Medición electrónica, ruido eléctrico, amplificador lock-in, trigonometría.

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The use of trigonometric identities is an important aspect in mathematics, sciences and engineering courses. Often become important to talk students about possible applications of the problems they solve in class rooms. This can serve as motivation and to make them familiar with the fundamentals of the instruments and the phenomena that they can find in their surrounding world. Thus, here we will briefly show how the solution of such a problem should be a good exercise to anticipate the notion of Lock-in detection since introductory trigonometry courses. This is important because we have witnessed in the last years the widespread and routine use of the so-called Lock-in amplifiers (LIA) in research and educational laboratories to detect and measure very small and noisy AC signals, therefore it becomes necessary to familiarize physics and engineering students since the first years of learning in high-schools, colleges and universities with their basic principle of operation, in the same way as they become familiar with other more traditional electrical signals measurement instruments such as galvanometers, conventional voltmeters, oscilloscopes and so on.

A LIA can be defined as an instrument that permits the measurement of AC signals modulated at a given frequency, *f*, in an environment of very low signal to noise ratio. The main function of LIA is then to measure the component of the signal at *f* and reject noise signals at other frequencies. Thus, in addition to the signal input one also needs to provide a LIA with a reference input containing a wave form (often a sinusoidal wave) at the reference frequency to "lock in" the response from an experiment at this frequency and to ignore anything that is not synchronized with it.

Although several experiments have been proposed to include this instrument in undergraduate experimental courses [1, 2, 3, 4, 5, 6], perhaps one of the most simplest exercise that can be implemented to demonstrate the advantages of lock-in detection is to modulate a light emitting diode (LED) with a square wave using a simple signal generator or a mechanical chopper, and detect the light output with a photodiode (PD). With a LED close to the PD the output signal from the PD can be clearly seen and can be directly measured on an oscilloscope screen. But when the LED is separated from the PD at a sufficient large distance, the trace on the oscilloscope disappears due to the noise introduced by the ambient light or due to the low impinging light intensity; however the signal remains strong if it is measured with a LIA.

Fig. 1 is a simplified block diagram illustrating the LIA operation. Although good tutorials to learn more about LIA can be found elsewhere [7], hopefully the approach followed here will allows introducing people into the basic principles behind this instrument in a simple but straightforward manner.

Suppose that the signal to be measured, $S(t)$, which is fed to the LIA input (see Fig. 1), has the form:

$$S(t)=A\cos(\omega t+\varphi)+n, \tag{1}$$

where $A$ is the signal amplitude, $\varphi$ is the phase, $\omega=2\pi f$ and $n$ represents the noise at the frequency $f$ (noise at other frequencies can be eliminated partially at an early stage, not shown in the schema, by using a convenient band-pass filter). Signal detection means then the measurement of its amplitude and phase. Suppose also that a reference signal at $f$ is fed to the reference input channel so that two waves can be generated with the same frequency: $r(t)=2\cos(\omega t)$ and $r'(t)=2\sin(\omega t)$. A multiplier circuit makes the product of the signal $S(t)$ and both the $r(t)$ and $r'(t)$ waves. Thus the output signals of the multiplier circuit are $p(t)=S(t)\times r(t)$ and $p'(t)=S(t)\times r'(t)$, respectively.

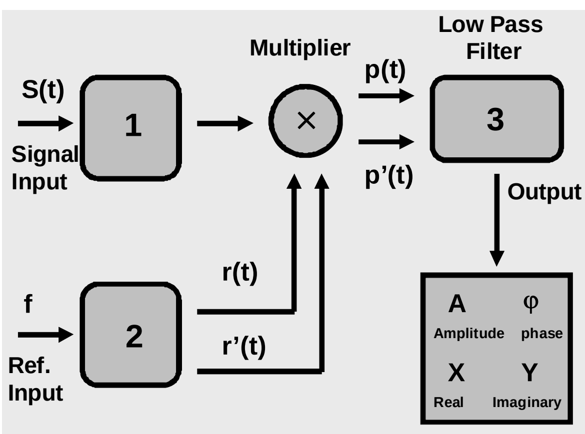


**FIGURE 1.** Block diagram to illustrate the operation principle of a LIA. The input signal $S(t)$ contains the noise. It is usually filtered to remove noise above and below the reference frequency by the device represented by Box 1 (actually a band-pass filter). The reference input signal is fed to the Box 2, a device which produces the two reference waves named $r(t)$ and $r'(t)$ (see text), which are used by the multiplier circuit to multiply the signal $S(t)$ and to obtain $P(t)$ and $P'(t)$. When these output signals pass trough Box 3 (a narrow enough low pass filter), their DC components $X=A\cos(\varphi)$ and $Y=A\sin(\varphi)$ remain. In this way the LIA can recover the real and imaginary components and from these the amplitude and phase of the analysed signal.

To calculate these products one can make use of some of the most useful typical trigonometric identities, namely:

$$\cos(a \pm b)=\cos(a)\cos(b) \mp \sin(a)\sin(b), \tag{2}$$

and

$$\sin(a \pm b)=\sin(a)\cos(b) \pm \cos(a)\sin(b). \tag{3}$$

They lead to:

$$2\cos(a)\cos(b)=\cos(a+b) + \cos(a-b), \tag{4}$$

and

$$2\sin(a)\cos(b)=\sin(a+b) + \sin(a-b), \tag{5}$$

respectively.

One can demonstrate after a straightforward calculation based in these equations that:

$$p(t)=S(t)\times r(t)=A\cos(\varphi)+A\cos(2\omega t+\varphi)+2n\cos(\omega t), \tag{6}$$

and

$$p'(t)=S(t)\times r'(t)=A\sin(\varphi)+A\sin(2\omega t+\varphi)+2n\sin(\omega t). \tag{7}$$

The Eqs. (6) and (7) contain frequency independent components, namely $A\cos(\varphi)$ and $A\sin(\varphi)$, the only that will remain if the signals $p(t)$ and $p'(t)$ are filtered with a device that obstruct the passage of the other terms (This can be realized in praxis using a so-called low pass filter. It can be shown that as long as there is no consistent phase relationship between the noise and the signal, the output of the multiplier due to the noise voltages will not be steady and can therefore be removed by the filter [8]).

Thus, this is in a few words the basic principle behind the operation of a LIA: From the input signal $S(t)$, and knowing the reference frequency, the function of a LIA is to recover $X=A\cos(\varphi)$ and $Y= A\sin(\varphi)$, from which the amplitude and phase can be determined as $A=(X^2 + Y^2)^{1/2}$ and $\varphi = \arctan(Y/X)$ respectively (note that the demonstration of these last equations also involve trigonometric algebra). If students have had an approach to complex numbers one can also use this exercise to recall them that $X$ and $Y$ are the real and imaginary parts of the complex number $A\exp(i\varphi)$, where $i=(-1)^{1/2}$.

In resume, a mathematics class room's exercise consisting in demonstrating the results given by Eqs. (6) and (7), following a simple trigonometric analysis that starts with the well known identities (2) and (3), can be used to provide teachers and students with a simple approach leading to the basic principles of LIA. They can go deeper inside this theme in more advanced intermediate level laboratory and/or electronic instrumentation courses. We feel that the approach presented here is plausible and satisfying to introduce and motivate students in LIA scientific instrumentation without involving excessively sophisticated mathematical analysis and/or electronic techniques. This approach has been used successfully during the lectures on Photothermal Techniques given by one of the authors (EM) at CICATA-IPN.

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[7] See for example the Instruction Manuals accompanying the Lock-in amplifiers from Stanford Research Instruments (http://www.thinksrs.com).

[8] Perkin Elmer Technical Note 1000 (http://cpm.uncc.edu/programs/tn1000.pdf), Visited 20 August 2009.