Doppler Effect of Mechanical Waves and Light

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Abstract

We discussed the Doppler Effect of mechanical waves when the relative velocity is not in the direction of wave vector; and we found that the observed frequency changes with time, which is different from the results when the relative velocity is along the wave vector direction. We showed a simple derivation of Doppler Effect equation for the light by using time dilation principle and showed that the motion of light source and observer has the same effect on the frequency shift.

Keywords: Doppler Effect, sound, light, time dilation, wave vector.

Resumen

El presente artículo presenta una discusión del Efecto Doppler en ondas mecánicas cuando se presenta una velocidad relativa que no se encuentra en dirección del vector de onda; se encontró que las frecuencias observadas varían respecto al tiempo, lo cual es diferente de los resultados respecto a la velocidad relativa a lo largo de la dirección del vector de onda. Se muestra una variación simple de la ecuación del efecto Doppler para la luz usando el principio de dilatación y mostrando que el movimiento de la fuente de luz y el observador presentan el mismo efecto en el cambio de frecuencia.

Palabras clave: Efecto Doppler, sonido, luz, dilación temporal, vector de onda.

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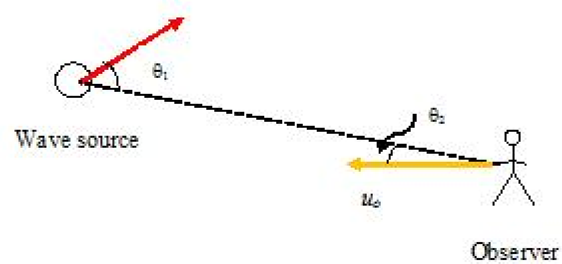
That the frequency measured by an observer may be different from that emitted from the wave source due to the relative motion between them is called Doppler Effect. For the mechanical waves (we simply call them sound in the following text), classical treatment can give satisfied result that has been given by many textbooks and discussed by Shaofu Wang [1]. It is

$$f_o = \frac{1+\frac{u_o}{v}}{1-\frac{u_s}{v}} f_s. \quad (1)$$

Where u_o, u_s, and v are the speeds of observer, sound source, and sound in the medium respectively; f_o and f_s are the frequency measured by the observer and emitted by the sound source respectively. When the source and observer are approaching to each other, u_o and u_s are positive; otherwise, they are negative. One should notice that the motion of observer and or source has different effect on frequency change.

Equation (1) is only valid when the relative velocity is along the line connecting the wave source and the observer (which is called wave vector). When this is not the case, u_o and u_s should be replaced with their velocity components that are parallel to the wave vector, as shown in Fig. 1. Therefore, Eq. (1) is replaced with

$$f_o = \frac{1+\frac{u_o\cos\theta_2}{v}}{1-\frac{u_s\cos\theta_1}{v}} f_s\ . \quad (2)$$



FIGUE 1. Situation that velocities are not along the wave vector.

Equation (2) shows: (a). For sound, only the velocity component along the wave vector direction affects the frequency. This is so called longitudinal Doppler Effect. (b) When observer and source move perpendicularly to the wave vector direction (transversely), *i.e.*, $\theta_1 = \theta_2 = 90°$, then $f_o = f_s$. So there is no so called transverse Doppler Effect for sound waves. (c) In the case that the wave source and observer are approaching/receding to each other, f_o actually changes with time because the angles change with time. In Fig. 2(a), we show the change of frequency ratio, f_o/f_s, as a function of time when either observer moves with u_o = 100 m/s and θ_2 = 30° toward the source or source moves with u_s = 100 m/s and θ_1 = 30° toward the observer. In Fig. 2(b), we set the angles $\theta_1 = \theta_2$ = 0°, which means that the source and observer move along the wave vector direction. We can see that f/f_s keeps constant before and after the source and observer pass each other; and there is a discontinuous change of f/f_s at the moment they meet.

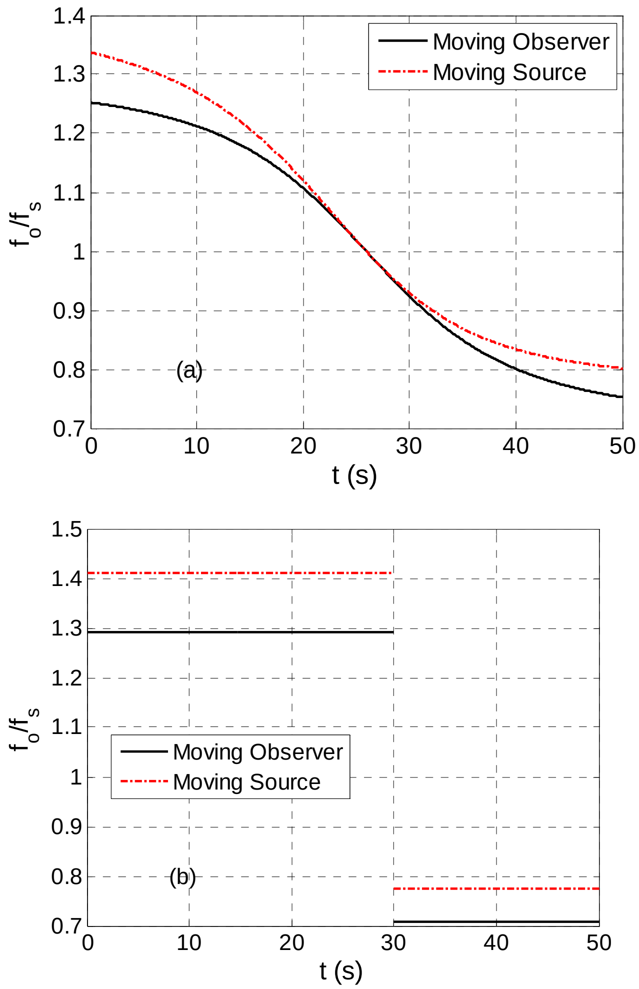


FIGURE 2. The frequency ratio f_o/f_s changes with time. (a) Two curves represent two scenarios: 1) Initial velocity of observer u_o = 100 m/s while the source is stationary, and initial angle θ_2 = 30° (black, solid line); the initial distance between the observer and source is set to be 3000 m. 2) The initial distance between the observer and source is still 3000 m, but now the initial velocity of the source u_s = 100 m/s while the observer is stationary, and initial angle θ_1 = 30° (red, dotted line). (b) While the other parameters keep unchanged, the initial angles are set as $\theta_1 = \theta_2$ = 0°. A sharp, discontinuous change of frequency appears at the moment that source and observer meet.

For the electromagnetic waves (we simply call them light in the following text) we have similar Doppler effect but the different equation because 1) propagation of light doesn't need medium so the light velocity relative to the observer is always the same; 2) the period of the light may change for the observers at different initial frame of reference (and time dilation). We start the discussion from the simplest case where the velocities of observer and source are along the wave vector between source and observer.

A. Moving observer

Shown in Fig. 3, the observer moves with velocity v toward the light source. In the inertial frame of reference of light source, K, the time for the observer needs to pass two crests (one wavelength) is (the period in K)

$$T = \frac{\lambda_s}{c+v} = \frac{1}{\left(1+\frac{v}{c}\right)f_s}. \qquad (3)$$

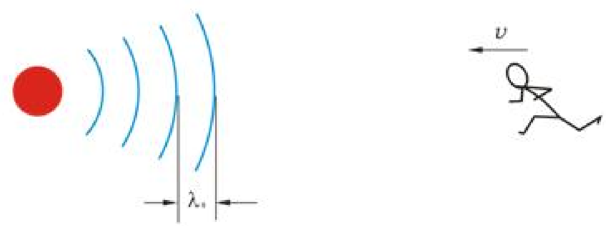


FIGURE 3. Stationary light sources and moving observer.

where c is the speed of light. If this period, T, is the same as T_o that measured in the frame of the observer, K', then this is exactly the same Doppler Effect equation as in sound waves. However, because the time dilation, the time measured by the observer is

$$T_O = \frac{T}{\gamma} = \frac{1}{\gamma\left(1+\frac{v}{c}\right)f_S}, \qquad (4)$$

where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, so
tT

$$T_o = \frac{1}{f_s}\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}},$$

and corresponding frequency is

$$f_o = f_s\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}. \qquad (5)$$

B. Moving light source

In the case that the light source moves with velocity v toward the observer, then in the frame of reference of the observer, K', the time interval between emission of successive wave crests, T, is different from the time interval between the arrival of successive crests because the crests are emitted at different locations. Therefore, the frequency measured by the observer, f_o, is not $1/T$. During the time T the crests ahead of the source move a distance

cT, and the source moves a shorter distance vT in the same direction. So the wavelength becomes $\lambda = (c\text{-}v)T$, as measured in the observer's frame of reference. The frequency that he measures is $f_o = c/\lambda$. That is

$$f_o = \frac{c}{\lambda} = \frac{c}{(c-v)T} = \frac{1}{(1-v/c)T} \,. \quad (6)$$

Still, if the period T measured by the observer were identical to the period, T_s, measured in the frame of reference of the light source, we would have ended up with the same Doppler Effect equation as shown in the Eq. (1). However, because the time dilation the transformation between T_s and T is

$$T = \gamma T_s = \frac{T_s}{\sqrt{1-v^2/c^2}} = \frac{1}{f_s\sqrt{1-v^2/c^2}} \,. \quad (7)$$

Substituting Eq. (7) to Eq. (6) yields Eq. (5) again. Therefore, with the light, unlike the sound, there is no distinction between motion of source and motion of observer; only the relative velocity of the two matters.

We should mention that in Eq. (3) we use classical velocity addition, $c+v$, to get the velocity between the observer and light, that is greater than the speed of light. It does not conflict with postulate in Einstein's theory of relativity which is: The speed of light is finite and independent of the motion of its source. (Another version is: In every inertial frame, there is a finite universal limiting speed c for physical quantity) [2]. This is because in this paper, we say that the speed of light is c, the speed of the observer is v; none of them exceeds the speed of light c; and the speed $c+v$ is not a speed of anything so it can be greater than c.

Usually $v \ll c$, so Eq. (5) can be expanded to Taylor series and to the first order, it is

$$f_o = f_s\left(1+\frac{v}{c}\right) \,. \quad (8)$$

Obviously, Eq. (8) shows that when the speed is low compared to c, measured frequency is proportional to the relative speed. If both f_o and f_s are known, Eq. (5) or (8) can be used to calculate the relative speed, which is the working principle of Doppler radar. The advantage of Doppler radar is that it can not only measure the position, but also the velocity of the object. In Fig. 4, we show how the measured frequency changes with relative velocity.

Let us take a look at an example. Suppose the emitting frequency of a radar is 2.7 GHz. A cloud moves toward the radar with speed of 30 m/s. what is the frequency detected by the radar?

There are two Doppler effects in this problem. The first one is the frequency, $f\,'$, 'seen' by the cloud. So the difference is

$$f' - f_s = f_s\frac{v}{c} = \frac{2.7\times10^9\times30}{3\times10^8}\ \text{Hz} = 270\ \ \text{Hz}\,.$$

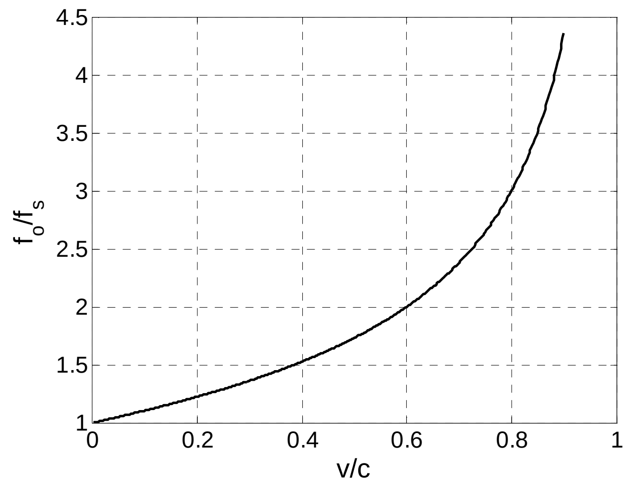


FIGURE 4. Change of ratio of f_o/f_s vs. velocity ratio v/c. When v/c is low, f_o/f_s is proportional to v/c; when v/c approaches to 1, f_o/f_s increases drastically with no upper limit.

Now the cloud becomes the light source with frequency $f\,'$ (reflected by the cloud). Suppose the frequency measured by the radar is $f\,''$, then

$$f'' - f' = \frac{f'v}{c} = \frac{(2.7\times10^9+270)\times30}{3\times10^8} = 270\ \text{Hz}.$$

And therefore

$$f'' - f_s = 540\ \text{Hz}$$

We can see that the frequency shift is very small compared to the light frequency but detectable.

There is a beautiful and simple derivation of Doppler Effect equation in Jackson's textbook [2] when the light source's or observer's velocity is not along the wave vector. Its result is

$$f_o = \gamma f_s\left(1-\frac{v\cos\theta}{c}\right)\,, \quad (9)$$

where θ is the angle between the velocity v and wave vector. When $\theta = 0$, Eq. (9) simplifies to Eq. (5). But when $\theta = 90°$, $f_o = \gamma f_s$, observed frequency is still greater than that from the source. This is so called transverse Doppler frequency shift which has been observed using a precise resonance-absorption Mossbauer experiment [3].

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