Application of single-slit diffraction to measure Young's modulus

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Abstract

We demonstrate an application of the laser diffraction by a single-slit to measure the Young's modulus of the material of a wire. The standard Searle's apparatus is modified to exploit the sensitivity of the diffraction pattern for changes in slit widths of the order of laser wavelengths produced by micro elongations in the wire when a stress is applied to it. The experiment is performed using different lasers as well as for wires of different materials and produces results with an error of few percent.

Keywords: Young’s modulus, single-slit diffraction, micro-displacement.

Resumen

Demostramos una aplicación de la difracción de láser por una sola ranura para medir el módulo de Young del material de un cable. La norma de aparatos de Searle se ha modificado para explotar la sensibilidad del patrón de difracción de los cambios en la anchura de rendija de la orden de longitudes de onda del láser producido por elongaciones micro en el cable cuando se le aplica un esfuerzo. El experimento se realiza con láseres diferentes, así como para alambres de diferentes materiales y produce resultados con resultados con un error de bajo por ciento.

Palabras clave: Módulo de Young’s modulus, difracción de una sola rendija, micro-desplazamiento.

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I. INTRODUCTION

The Young's modulus of materials is a measure of its response to the applied stress. It is a very important quantity because it defines the rigidity of materials in practical applications [1]. Many methods exist and different types of instruments are available to measure the Young's modulus of the material of a wire [2, 3, 4, 5, 6, 7, 8]. Most of these methods directly measure the strain (small changes in the length of wire per unit length) when a stress is applied. For example, in the standard Searle's method the small elongation of the wire is measured using a spirit level and a compensating screw [1]. The elongation of the wire therefore should be large enough, within its elastic limit, to yield appreciable accuracy. This is generally achieved by using a long enough wire and larger loads. In addition, the traditional method suffers from the backlash error of the compensating screw. In order to measure the Young's modulus precisely one needs an alternative sensitive method to measure small elongations of the wire due to the applied stress.

It is well known that the diffraction of light by a single-slit is very sensitive to micro-changes in the slit widths, of the order of wavelengths of light ($\lambda \cong 632nm$ for red laser) [9]. This sensitivity has been used to measure small displacements, micro-sized biological samples of various shapes [9]. Here we aim to modify the Searle's apparatus to exploit this sensitivity of the single-slit diffraction pattern to measure the Young's modulus. In the following, we shall recall necessary theoretical background with a description of our apparatus followed by the results and conclusions.

II. THEORETICAL BACKGROUD

The Young's modulus (Y) of the material of a wire is a ratio of the longitudinal tensile stress to the tensile strain [1]. A force F is provided by suspending a mass M o the wire of radius r which causes an elongation e from its original length L. The expression for the Young's modulus is given by,

$$Y = \frac{MgL}{e\pi r^2} \qquad (1)$$

where g is the gravitational acceleration. In order to measure Y efficiently, one needs to have a sensitive technique to measure small elongations produced in the wire by the applied stress.

We apply the single-slit diffraction pattern as a sensitive tool to measure the micro-elongations of the order of wavelength of laser used to produce diffraction. The diffraction pattern by a narrow single-slit consists of a broad and intense central maximum and a large number of (high orders) symmetrically placed maxima and minima on either side (see Fig. 1 for schematics) [1, 9]. The intensity of far-field diffraction pattern is given by, $I = I_m(\sin\alpha/\alpha)^2$, with $\alpha = \pi d\sin\theta/\lambda$, where d is the distance between two sharp edges forming the slit (slit width), θ is an angle between incident and diffracted ray, and λ is wavelength of the light [9]. In the single-slit diffraction the first dark fringe (minima) are obtained when the path difference between the top and bottom rays is equal to λ. If the angle θ between the incident and diffracted ray is small ($\theta < 4^\circ$), we can replace

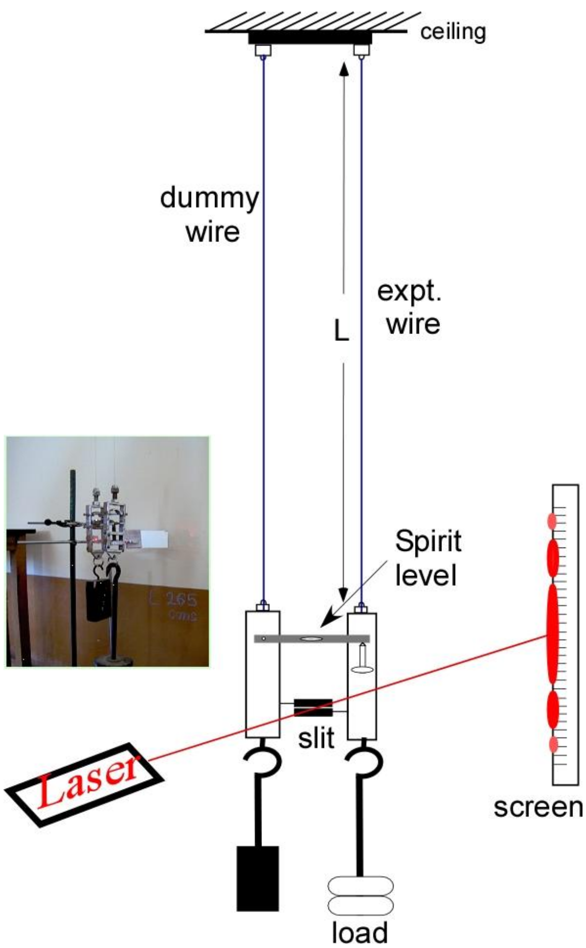

FIGURE 1. Schematic of the modified Searle's apparatus to measure the Y by laser diffraction. The inset shows a picture of the setup.

$\sin\theta \cong x/D$, where D is the distance between the slit and the screen and x is a position on the screen. The quantity of interest for measuring the Young's modulus is the spacing between the minima lying on either side of the central maximum,

$$\delta x = \frac{2\lambda D}{d} \qquad (2)$$

Note that δx is inversely proportional to the slit width d. The diffraction pattern can be magnified by choosing D sufficiently large so that the effect of micro-changes in the slit width becomes clearly noticeable on the screen [10, 11].

III. EXPERIMENTAL SETUP AND PROCEDURE

The standard Searle's apparatus is slightly modified to measure the elongation of the wire using the single slit laser diffraction (see Fig. 1 for schematics of the setup). Two sharp blades are attached to the two wires; one remains fixed with the reference wire, while the other one is movable with the experimental wire of the same

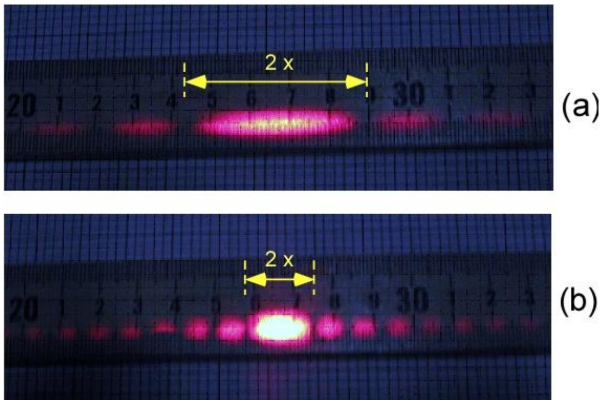

FIGURE 2. The experimental diffraction pattern seen on a screen with D=10m using red laser for two different loads.

is fixed with the reference wire, while the other one is movable with the experimental wire of the same material. These two blades are arranged in such a way that they form a narrow parallel single-slit. The diffraction pattern by the single slit is viewed on a far-off screen and the positions of the minima are recorded. As we increase the load on the experimental wire, the slit width changes due to the elongation in the experimental wire and therefore the diffraction pattern changes. The change in the positions of the minima is used to measure the elongation in the wire using Eq. (2). Compared to the Searle's method, the diffraction method is sensitive to small displacements of the order of laser wavelengths, and in particular it does not suffer from any screw backlash errors.

Before starting the experiment, two identical wires on the Searle's apparatus are kept under sufficient loads for some time. This removes any minor kinks and twists present in the wire. In the beginning, one should be careful not to choose a very heavy load on the experimental wire to exceed the elastic limit of the material. The movable blade attached to the experimental wire, is then adjusted so as to make a single parallel slit with the fixed blade. The red laser ($\lambda \cong 632nm$) then produces a clear diffraction pattern showing maxima and minima up to several higher orders as shown in Fig. 2 (a). This single slit diffraction pattern is projected on a screen placed about 10m away from the slit. Due to the high contrast of the diffraction pattern, we use a meter scale to measure the positions of the central maximum and the two first order minima on a graph paper fixed on the screen using digital Vernier callipers.

To produce the elongation in the wire we use calibrated weights. Two sets of measurements are taken, one by increasing the load (loading) and other by decreasing the load (unloading). The almost identical diffraction pattern observed for loading and unloading ensures that the wire follows the Hooke's law, *i.e.*, the stress is applied within its elastic limit. The distance between two neighboring first order minima δx, is used to calculate the increment in the slit width using the Eq. (2). Knowing the values of the slit widths for different loads, changes in the slit widths can be determined as a function of the load, which is a direct measure of the elongation of the experimental wire.

IV. RESULTS AND DISCUSSIONS

Using the red laser, a set of observation for mass against extension is taken with a chromel wire. In Fig. 3, we plot the mass versus elongation (e) for the chromel wire. The data shows a linear behavior and are fitted with a straight line to get the slope *e*/*M* of the line. From the measured value of e/M, we calculate value of the Young's modulus [using the Eq. (1)] to be $Y = 1.53 \times 10^{11} Nm^{-2}$ with an error of about 5%. Most of the error is due to the location of intensity minima and can be further improved. Furthermore, we have repeated these measurements for another green laser ($\lambda \cong 532nm$). Keeping all other conditions identical, the value $Y = 1.56 \times 10^{11} Nm^{-2}$ matches within few percent with the one obtained using the red laser. In order to cross check the value of Y obtained by diffraction method, we have measured Y using the traditional Searle's method also. The elongation of the wire is measured by compensating the displacement of bubble in the spirit level using a micro-screw. Note that under identical condition we have to use almost double loads to get measurable displacements with the bubble. The value of Y obtained is in agreement with the one obtained using diffraction method and it also agrees with the value quoted in the literature within few percent.

We have performed another set of experiments using the diffraction method to determine the value of Y for a tungsten wire. The curve for mass versus elongation for the diffraction method using red laser for the tungsten wire is shown in Fig. 4. As expected, these data points also exhibit a linear relationship and the obtained value for the Y agrees with the value quoted in the literature with 5% percent error.

All the measurements of Y are cross examined using the Searle's method and also with a green laser beam.

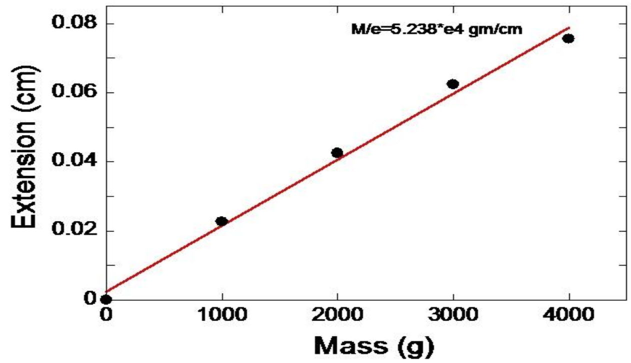

FIGURE 3. The Data for stress versus strain for cromel wire (L=1.5m, r=0.15mm). The line is a linear fit. Each data point is averaged over two values corresponding to loading and unloading sequence.

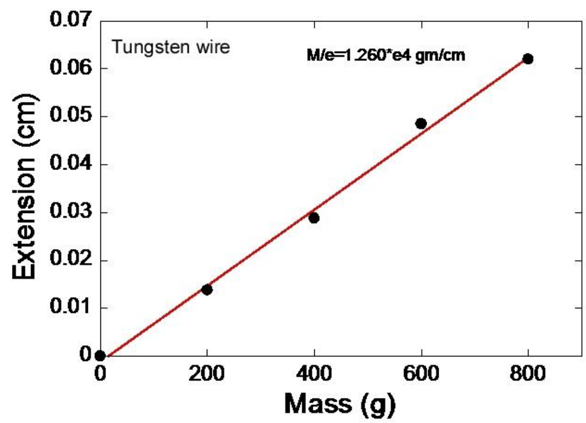

FIGURE 4. Same as in Fig. 3 but for the tungsten wire.

We would like to mention some remarks on performing the experiment correctly. Because the diffraction pattern is sensitive to the displacements of the order of laser wavelength, one needs to be careful about any minor kinks present in the wire. If the kinks are not removed prior to the measurement they can cause significant errors in the measurement, in particular, when using lighter loads. The diffraction pattern is also sensitive to any deformations in the shape of the slit. Therefore, the two blades must be kept as much parallel as possible and should remain so during the experiment. Prior to the experiment the parallelism of the side of the slits is verified experimentally, by ensuring identical symmetrical diffraction pattern when the laser beam is scanned horizontally along the entire slit opening. By measuring distances between first minima across the slit, the angular mismatch between parallel edges is estimated to be $\approx 3^{\circ}$. Once the slits are made parallel, they remain so because the elongation during the experiment does not change the slit orientation. Furthermore, minor misalignment in slits, however, would not affect our measurement since we illuminate around the same point on the slit opening during the experiment.

The main advantage in using the diffraction method is its higher sensitivity compared to the traditional one, and the fact that it does not suffer from any screw backlash errors which may be a dominating source of error in the traditional method. Further improvements in the experimental set up are possible. For example, one can use image analysis techniques to determine the precise location of the diffraction minima using a calibrated CCD and computer.

V. CONCLUSIONS

We have proposed a sensitive optical method to measure the Young's modulus of the material of a wire using the single slit diffraction. The apparatus is a modified Searle's apparatus, so as to incorporate a narrow parallel slit created by blades attached to the two wires. The change in the elongation in the experimental wire directly changes the slit-opening, which is measured by the change in the positions of the minima in the diffraction pattern. The micro displacements of the order of laser wavelengths can be easily measured due to the sensitivity of the diffraction phenomenon. One can design a compact setup to measure Y since micron sized changes become measurable with the diffraction technique. It is also possible to determine other physical quantities of interest that involves measuring micro-increment like the thermal expansion coefficient a wire or a rigid body.

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