Natural cooling of hot water: An experimental study in thermal processes

Vasudeva Rao Aravind[1], Mehmet Goksu[2], Hope Miller[1] and John W. Heard[1]

[1]*Physics Department, Clarion University of Pennsylvania, Clarion, PA 16214, USA.*
[2]*Physics Department, Millersville University, Millersville, PA 17551, USA.*

E-mail: varavind@clarion.edu

(Received 29 July 2010, accepted 5 September 2010)

Abstract

Experiments in thermodynamics and heat transfer are common in any undergraduate introductory physics laboratory course. In this paper, we discuss an experiment to quantify thermal conduction coefficient of objects using simple laboratory tools. We experimentally observe the temperature drop of a hot cup of water placed in a closed container in ambient room temperature. By observing this temperature change, and by carefully modeling the heat losses from various parts of the container, we show that we can extract thermal conduction coefficient of the material of the cup, with reasonable accuracy.

Keywords: Thermodynamics, heat transfer, introductory physics

Resumen

Los experimentos de termodinámica y transferencia de calor son comunes en cualquier curso introductorio de pregrado de laboratorio de física. En este trabajo, se discute un experimento para cuantificar el coeficiente de conducción térmica de los objetos utilizando herramientas sencillas de laboratorio. Observamos experimentalmente el descenso de la temperatura de una taza de agua colocada en un recipiente cerrado a temperatura ambiente. Observando este cambio de temperatura, y mediante una cuidadosa modelización las pérdidas de calor de varias partes del contenedor, se muestra que podemos extraer el coeficiente de conducción térmica del material de la copa, con una exactitud razonable.

Palabras clave: Termodinámica, transferencia de calor, física introductoria.

PACS: 01.50.H-, 01.40.-d, 01.50.Pa **ISSN 1870-9095**

I. INTRODUCTION

When heat is transferred to or from a substance, the internal energy of the substance can change. Transfer of heat affects us in many ways. For instance, the furnaces in our homes distribute heat on cold days, and air conditioners remove heat on hot days. Our bodies constantly transfer heat in one direction or another, to prevent the effects of hypo- and hyperthermia [1].

Heat transfer to or from a material can happen through different means termed as conduction, convection and radiation. Conduction is the process whereby heat is transferred directly through a material, without any bulk motion of the material playing a role in the heat transfer. Convection is the process in which heat is carried from one place to another by the bulk movement of a fluid. Radiation is the process in which energy is transferred by means of electromagnetic waves, without the need for a physical contact between transmitting and receiving medium [2, 3].

In this paper we will study the heat loss by conduction from a hot cup of water, made of Styrofoam. When a hot cup of water is placed in contact with surrounding air, the water loses heat by conduction through the walls of the cup and convection between surrounding air and the wall of the cup.

II. THEORY

As mentioned above, there are three heat transfer mechanisms: conduction, convection, and radiation. In this paper, we assume that the water loses heat purely by conduction. Conduction can be considered to be a direct flow of heat through two ‘touching’ systems. If a ‘hot’ object is kept in contact with a ‘cold’ object, the rate of heat transfer is given by the equation:

$$\frac{\Delta Q}{\Delta t} = -k_t A \frac{|\Delta T|}{\Delta x}, \qquad (1)$$

where ΔQ is the energy lost or gained, ΔT is the time interval, k_t is called the 'thermal conductivity' of the material, A is the area through which the heat is conducting, Δx is the thickness of the material, and ΔT is the change in temperature driving the conduction.

For a small system, such as a coffee cup, heat flowing into or out of it will change its temperature, which will in turn change the heat flow rate because the heat can be written calorically as:

$$Q = mc\,\Delta T. \qquad (2)$$

This means we can write the differential change in temperature. Note that 'ΔT' in equations (1) and (2) have different meanings. In equation (1) it means the temperature difference between the hot water and air while in equation (2) it represents the hot water temperature change for a time interval of ΔT.

If we call dT the infinitesimally small temperature difference ($\Delta T \to 0$) in an infinitesimally small time interval dT ($\Delta T \to 0$), then we can write, from equations (1) and (2),

$$\frac{dT}{T - T_0} = -\left(\frac{k_t A}{mc\Delta x}\right) dt, \qquad (3)$$

where T is the temperature of the water at any time t and T_0 is the initial temperature of the hot water, at time t=0. Equation (3) can be solved by calculus [4] to yield the temperature at any time t:

$$T(t) = T_A - T_0 \; e^{-Ct} + T_0, \qquad (4)$$

where $C = \left(\frac{k_t A}{mc\Delta x}\right)$ is obtained fitting experimental data to equation (4).

III. EXPERIMENT

We experimentally observed the cooling of a hot cup of water at room temperature. Figure 1 shows the experimental setup for this measurement. The cup was made of styrofoam and the temperature as a function of time [$T(t)$] was observed with the help of a Vernier temperature sensor, interfaced with a computer using 'Logger Pro' software [5].

An identical cup was inserted into the cup of water, so that the hot water was surrounded by Styrofoam on all sides (as shown in figure 2). This minimizes the error due to convective heat loss by direct contact with air.

In order to model the heat losses from the water by conduction, we need to pay careful attention to the various surfaces through which heat can be lost. Figure 2 shows a schematic diagram of the cup, illustrating different parameters associated with the heat transfer.

FIGURE 1. Experimental setup to observe the cooling of a hot cup of water. Note that the an identical cup was inserted into the cup of water, so that errors due to convective heat loss by direct contact with air can be minimized.

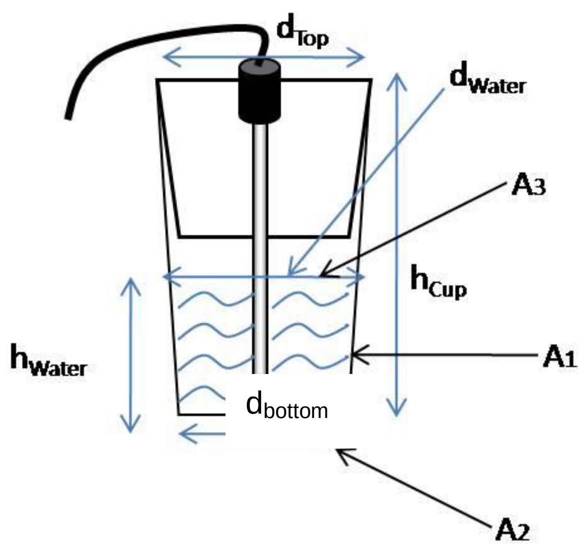

FIGURE 2. Schematic diagram showing the picture of the cup of hot water through which heat is lost. The immersed object in the middle of the cup is the temperature sensor interfaced with computer.

We first of all notice that the cup of water may not be in any regular shape. We will need to make some approximations to understand the experimental results. We denote d_{top} to be the diameter of the top of the cup, and d_{bottom} to be diameter of the bottom of the cup. Height of the cup is denoted as h_{cup} and height of the water level in the cup is denoted as h_{water}. Different surfaces that can contribute to conductive heat loss are: 1. Area of the curved surfaced of the cup, 2. Bottom surface of water level in the cup and 3. top of the cup. These areas are labeled A_1, A_2 and A_3 respectively.

In order to determine the diameter of water on the top of the cup, we assume the diameter of water is minimum at the bottom and then increases linearly all the way to the top. This assumption is reasonable, considering the fact that the cross section of the curved surface of the cup is linear. So we define the diameter of top level of the water as:

$$d_{water} = d_{bottom} + \left(\frac{h_{water}}{h_{cup}}\right) \times \; d_{top} - d_{bottom} \; . \qquad (5)$$

To find the area of the curved surface of the cup, we approximate the water-containing portion of the cup to be a cylinder with an average diameter of $d_{water} + d_{bottom} \;\; / 2$ and height h_{water}. So we have:

$$A_1 = \pi \, h_{water} (d_{water} + d_{bottom}) / 2, \qquad (6)$$

$$A_2 = \pi \left(\frac{d_{bottom}}{2}\right)^2 , \qquad (7)$$

$$\text{and } A_3 = \pi \left(\frac{d_{water}}{2}\right)^2 . \qquad (8)$$

The total area A through which the water loses heat by conduction is given by $A=A_1+A_2+A_3$.

Figure 3 shows the results of the experiments. The temperature of the water in the cup decreases exponentially with time. The exponent can be determined by fitting this exponential decay with equation (4).

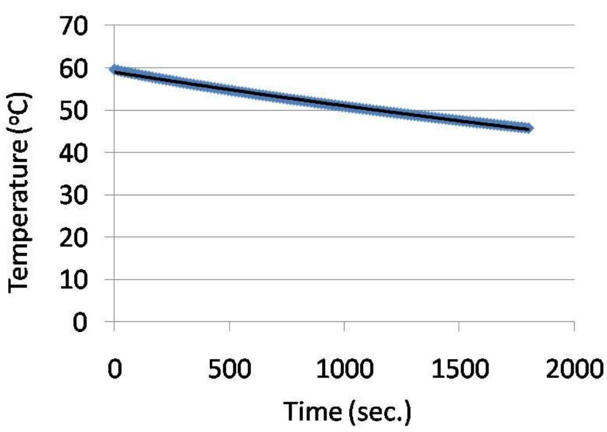

FIGURE 3. Experimental observation of the cooling of water. The blue data points are experimental data and thick black line is the fit to equation (4).

In our experiment, the parameters were: mass of water = 0.0882 kg, specific heat of water (from literature), c=4186J/(kg.^{o}C), thickness of the cup, Δx=0.002 meters, total area of styrofoam in contact with air = 0.0188 m^2 (including the top, bottom and curved surface of the cup).

By fitting this experimental data to equation (4), we determined that the fitting exponential coefficient, C=0.00026 sec^{-1}. From this value, we can determine the thermal conductivity of styrofoam by rewriting the expression for the fitting coefficient C,

$$k_t = \frac{mc\Delta x C}{A}. \qquad (9)$$

From our experiments, we find that the calculated value for thermal conductivity of Styrofoam is 1.0×10^{-2} W/(m.K).

IV. DISCUSSION

The thermal conductivity of Styrofoam available from literature is 0.033W/(m.K) [6]. We first of all note that using the approximations that we have used, by modeling the heat loss from a hot cup of water, we were first of all able to get an order-of-magnitude value for the thermal conductivity of Styrofoam. We have not accounted for the convective and radiative heat loss from the water, which explains the discrepancy between measured and literature values.

As a future work, by considering the convective heat transfer between the surface of the cup and the surrounding environment, and by using a material with well known thermal conductivity, one can measure the convective heat transfer coefficient between the cup and the surroundings.

ACKNOWLEDGEMENTS

Vasudeva Rao Aravind and Hope Miller acknowledge the financial support received from *Clarion University Undergraduate Research, Scholarly, Creative, Entrepreneurial and Civic Projects Grant*. We also acknowledge the financial support for the project from the Physics Department, School of Arts and Sciences, Clarion University. Mehmet Goksu acknowledges the financial support from Physics Department, Millersville University.

REFERENCES

[1] Cutnell, J. D., Johnson, K. W., and Fischer, K. D., *Physics* (John Wiley and Sons, Inc., 2008).

[2] Serway, R. A., and Jewett Jr., J. W., *Physics for Scientists and Engineers* (Brooks/Cole Cengage Learning, 2010).

[3] Walker, J., *Fundamentals of Physics* (John Wiley and Sons, Inc., 2008).

[4] Edwards, H. C., and Penney, D. E., *Calculus with analytical geometry* (Prentice Hall, New Jersey, 1998).
[5] http://www.vernier.com/soft/lp.html (retrieved July 29, 2010).
[6] http://www.engineeringtoolbox.com/thermal-conductivity-d_429.html (retrieved July 29, 2010).