Special Relativity and textbook exercises

Julio Güémez
Departamento de Física Aplicada, Universidad de Cantabria, Spain.

E-mail: guemezj@unican.es

(Received 8 June, 2011, accepted 17 September 2011)

Abstract
A textbook exercise about a block pulled on a rough surface by a conservative force and against a dissipative frictional force is described using a 4-vector relativistic thermodynamics approach, taking into account thermal effects produced in diathermal contact with a heat reservoir. The conservative force is described using a relativistic electromagnetic formalism and the whole exercise is solved in a Lorentz covariant form, first in 'privileged frame' S_∞ in which both surface and heat reservoir are at rest, and after that in frame S_A, in standard configuration with respect to frame S_∞. This exercise could by of interest to undergraduate students with an interest in Special Relativity and Thermodynamics.

Keywords: Special Relativity, four-vectors, relativistic thermodynamics first law.

Resumen
Se describe un ejercicio de libro de texto, un bloque arrastrado sobre una superficie rugosa mediante una fuerza conservativa y contra una fuerza de fricción, utilizando un formalismo de 4-vectores de termodinámica relativista, considerando los efectos térmicos que se producen en el contacto diatermo del bloque con un foco térmico. La fuerza conservativa se describe utilizando un formalismo electromagnético relativista y el ejercicio completo se resuelve en forma covariante Lorentz, primero en el sistema de referencia 'privilegiado' S_∞ en el que tanto la superficie como el foco térmico se encuentran en reposo, y después en el sistema de referencia S_A, en configuración estándar con respecto a S_∞. Este ejercicio puede ser atractivo para aquellos estudiantes universitarios con interés en la Relatividad Especial y en la Termodinámica.

Palabras clave: Relatividad Especial, cuadrivectores, primer principio de la termodinámica relativista.

PACS: 05.70.-a, 03.30.+p, 01.55. + b **ISSN 1870-9095**

I. INTRODUCTION

Introductory physics textbooks consider exercises in which a solid block moves on a rough surface with a friction force applied to it (Fig. 1) ([1], p. 198). A block Z, with mass *m*, is accelerated from rest along a table by a force *F* (all forces are one-dimensional and vectorial notation is dropped) applied to it and against a sliding friction force *f*. It is found by experiment that a portion of the table along which the block slides becomes warmer ([2], pp. 153-154). The thermal effects of friction forces are rarely formalized in university textbooks ([3], pp. 204-207).

It is always interesting to ask for the description of the same process from the point of view of an observer that moves with constant speed with respect to the 'privileged frame' S_∞ (lab frame) [4]. Galileo's Principle of Relativity is rarely applied to textbook exercises that include thermal effects.

On the other hand, university physics textbooks do not solve this kind of exercise either using concepts of special relativity ([5], pp. 359, 361) or Minkowski's 4-vector formalism [6]. This absence of special relativity and 4-vector formalism in the resolution of exercises in university physics textbooks is a pedagogical shortcoming [7].

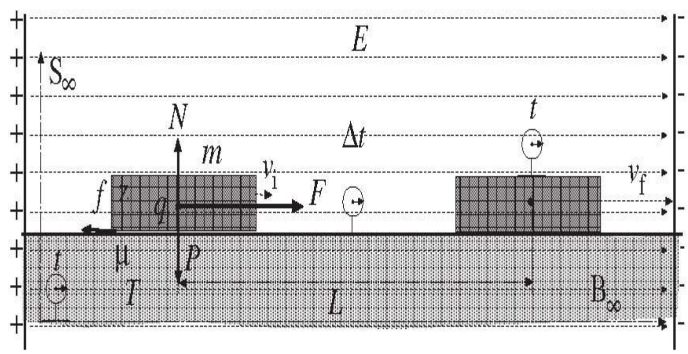

FIGURE 1. A block Z, with an electric charge *q* fixed in it, is located inside a horizontal electric field $\mathcal{E}$ produced between the plates of a parallel-plates charged capacitor C. A constant horizontal force $F = q\mathcal{E}$ pulls Z along the *x* axis and a constant kinetic frictional force *f* –with friction constant μ – opposes *F*. Forces are applied during a time interval $\Delta t = t_0$. Body Z center of-mass displacement is *L* and it varies its velocity from v_i to v_f. The large table B_∞ – a heat reservoir at temperature *T* – and the capacitor C are at rest in reference frame S_∞. Clocks, at rest or in movement, exhibit a universal time *t*.

When an exercise is solved in the relativistic Einstein-Minkowski 4-vector formalism, the solution is Lorentz invariant, in contrast with the classical solution, which is not [8]. Moreover, the Newton-Galileo solution is obtained in the limit in which $c \to \infty$.

In contrast to what happens in university textbooks, the special theory of relativity continues to arouse great interest among both teachers [9] and students, as is evidenced by the important number of articles on this topic [10] published during the past few years in journals devoted to undergraduate physics education [11].

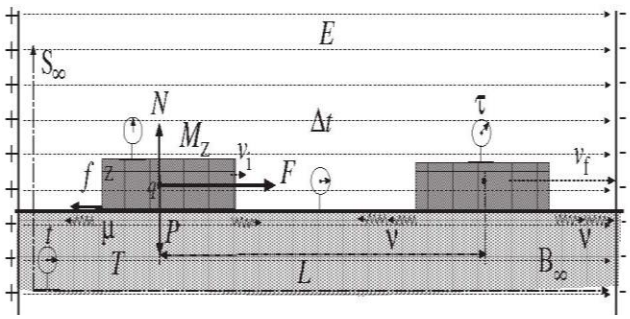

FIGURE 2. Relativistic description of the situation in Fig. 1. Clocks at rest in S_∞ indicates its time t. Body Z proper time τ is measured by a clock attached to it [11]. Thermal energy is characterized as photons with frequency ν absorbed by the heat reservoir B_∞.

II. RELATIVISTIC THERMODYNAMICS FIRST LAW

Consider the relativistic description of the process given in Fig. 2. A rigid (see Appendix) body Z with constant inertia M_Z (see Appendix) which moves with initial velocity v_i in frame S_∞, has linear momentum $p_i = \gamma(vi)M_Z v_i$ and total energy $E_i = \gamma(v_i)M_Z c^2$ [12], where $\gamma(v) = (1 - v^2/c^2)^{-1/2}$, and its initial 4-vector [13] energy function [14] U_i^μ is (for typographical reasons, a contravariant column 4-vector is expressed as a row 4-vector, maintaining its contravariant Greek index):

$$U_i^\mu = \left\{c\gamma(v_i)M_Z v_i, 0, 0, \gamma(v_i)M_Z c^2\right\}. \tag{1}$$

For a finite process, during which different forces are applied to Z during a time interval t_0, it reaches a final velocity v_f , and its 4-vector final energy function U_f^μ is:

$$U_f^\mu = \left\{c\gamma(v_f)M_Z v_f, 0, 0, \gamma(v_f)M_Z c^2\right\}. \tag{2}$$

For a conservative force $F = q\mathcal{E}$ with associated 4-vector displacement $L^\mu = \{L, 0, 0, ct_0\}$, L being the displacement of its centre-of-mas, the 4-vector work W_F^μ is (see Appendix):

$$W_F^\mu = \{cqEt_0, 0, 0, qEL\}. \tag{3}$$

This 4-vector W_F^μ is expressed as the (minus) increment in energy function of a work reservoir (the battery that provides the electric charge on capacitor and the electric field), with $W_F^\mu = -\Delta U_F^\mu$.[15].

In a process in which body Z slides on the surface of the quasi-infinite inertia system B_∞ [4], the force f is described using two phenomenological rules ([16], pp. 6-7): Amonton's rule, where the frictional force f is proportional to the normal N holding the two surfaces, and it is independent of the area of the surfaces in contact, and Coulomb's rule, where force f is independent of the sliding velocity ([17], pp. 884-885). We assume that force f is described in special relativity using these rules, with: $f = -\mu M_Z g$, where $N = M_Z g$. A friction force has no associated displacement in frame S_∞, in which B_∞ remains at rest ([18], p. 616), its 'product force-displacement' [19] is null and it is a zero-work force [20]. The 4-vector work W_F^μ for f is:

$$W_f^\mu = \{-c\mu M_Z g t_0, 0, 0, 0\}. \tag{4}$$

The 4-vector W_F^μ cannot be expressed as the increment in energy function of a work reservoir.

For a solid body, forces are simultaneously applied to it in frame S_∞ during time interval t_0 [21]. The 4-vector total work ([22], p. 84): $W^\mu = W_F^\mu + W_f^\mu$ is:

$$W^\mu = \left\{c\left(q\varepsilon - \mu M_Z g\right)t_0, 0, 0, q\varepsilon L\right\}. \tag{5}$$

When a friction force acts during a certain mechanical process on a rigid body, like the process described in Fig. 2, mechanical energy is dissipated as heat. A complete description of such a process includes a relativistic description of heat [23], and its corresponding 4-vector thermal energy Q^μ.

Energy interchanged as heat is formed by photons [24] with total zero linear momentum (thermal photons) (see Appendix). A photon with frequency ν has energy $e_p = h\nu$. In the thermal photons monochromatic approximation [25] it is assumed that every interchanged photon has the same frequency. During a friction process, total energy $E_p = Nh\nu$ is interchanged as thermal energy, where N is the number of photons transferred. We take heat Q to be positive when heat is transferred to body Z and negative when the transfers are out of the body. With $Q = -Nh\nu$, the 4-vector heat Q^μ is given in frame S_∞ as [8]:

$$Q^\mu = \{0, 0, 0, -Nh\nu\}, \tag{6}$$

The inertia of this ensemble of photons [26] is $M_T = (Nh\nu)c^{-2}$ ([27], p. 232). The 4-vector Q^{μ}. can be expressed as the (minus) increment in energy function of a heat reservoir (the floor B_∞) at temperature T, with $Q^{\mu} = -\Delta U_T^{\mu}$ [15].

The Relativistic Thermodynamics First Law in frame S_∞ for the process under consideration is given by [8]:

$$U_f^{\mu} - U_i^{\mu} = W^{\mu} + Q^{\mu}. \tag{7}$$

According to our treatment of work for a conservative force ($W_F^{\mu} = -\Delta U_F$) and heat ($Q^{\mu} = -\Delta U_T$), the Relativistic First Law can be expressed as:

$$\Delta U^{\mu} + \Delta U_F^{\mu} + \Delta U_T^{\mu} = W_f^{\mu}. \tag{8}$$

This equation emphasizes the role played by the heat reservoir B_∞ as a thermal radiation sink, and the role as work reservoir of the battery that charges the capacitor C, which provides the work done on Z ([28], p. 290).

From Eq. (8) and the previously obtained 4-vectors, one obtains the equations:

$$\gamma(v_f)M_Z v_f - \gamma(v_i)M_Z v_i = \left(q\varepsilon - \mu M_Z g\right)t_0, \tag{9}$$

$$\gamma(v_f)M_Z c^2 - \gamma(v_i)M_Z c^2 = q\varepsilon L - Nh\nu. \tag{10}$$

Eq. (9) is the relativistic impulse-momentum equation (relativistic Newton's second law) for the process under consideration and Eq. (10) is the corresponding energy equation (relativistic energy conservation law). Taking into account that ([29], p. 37):

$$\frac{\mathrm{d}\left[\gamma(v)c^2\right]}{\mathrm{d}\left[\gamma(v)v\right]} = v = \frac{\mathrm{d}\,x}{\mathrm{d}\,t},$$

for a finite process with constant force F, applied during time interval t_0, and constant inertia M_Z, with the body's centre-of-mass displacement L:

$$\frac{M_Z\Delta\left[\gamma(v)c^2\right]}{M_Z\Delta\left[\gamma(v)v\right]} = \frac{FL}{Ft_0}.$$

When the impulse-momentum Eq. (9) is fulfilled then the relativistic centre-of-mass equation is fulfilled too, with:

$$\gamma(v_f)M_Z c^2 - \gamma(v_i)M_Z c^2 = \left(q\varepsilon - \mu M_Z g\right)L. \tag{11}$$

This relativistic centre-of-mass equation is expressed as $\Delta K_Z = F\Delta x_{cm}$, where K_z is the kinetic energy, F is the resultant force applied on Z and Δx_{cm} is its centre-of-mass displacement [30]. Comparing the energy equation Eq. (10) and the centre-of-mass equation, Eq. (11), one obtains the thermal energy equation:

$$Nh\nu = \mu M_Z gL, \tag{12}$$

with $\Delta U_T = \mu M_Z gL$.

A. Description in frame S_A

For an observer in frame reference S_A in standard configuration with respect to S_∞ [31], with velocity $\vec{V} = (V,0,0)$ with respect to frame S_∞, the Lorentz transformation matrix for the standard configuration is ([6], p. 236):

$$L_\nu^{\mu}(V) = \begin{Bmatrix} \gamma(V) & 0 & 0 & -\beta(V)\gamma(V) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta(V)\gamma(V) & 0 & 0 & \gamma(V) \end{Bmatrix}, \tag{13}$$

with $\beta(V) = V/c$ and $\gamma(V) = \left[1-\beta^2(V)\right]^{-1/2}$. When an exercise is solved in the frame S_∞ using 4-vector equations, the corresponding magnitudes (velocity, force, frequency, etc.,) in S_A are obtained through relativistic transformation (relativistic transformation of velocity, relativistic force transformation, Doppler effect, etc.) and the corresponding 4-vectors in S_A are obtained through the Lorentz transformation [32]. For example, from 4-vector initial velocity v_i^{μ}:

$$v_i^{\mu} = \left\{\gamma(v_i)v_i, 0, 0, \gamma(v_i)c\right\},$$

the corresponding 4-vector v_{iA}^{μ},

$$v_{iA}^{\mu} = L_\nu^{\mu}(V)v_i^{\nu} = \left\{\gamma(v_{iA})v_i, 0, 0, \gamma(v_{iA})c\right\},$$

implies that:

$$\gamma(v_A)v_A = \gamma(v)\gamma(V)(v-V), \tag{14}$$

$$\gamma(v_A) = \gamma(v)\gamma(V)(1 - vV/c^2), \tag{15}$$

and

$$v_{iA} = \frac{v_i - V}{1 - v_i V/c^2}.$$

Any other magnitude in S_A is obtained in a similar way [8].

The Lorentz transformation immediately provides the correct description in S_A for the process previously described in S_∞. For instance, with

$$W_{FA}^{\mu} = L_{\nu}^{\mu}(V)W_F^{\nu} = \left\{c\gamma(V)qE\left(t_0 - \frac{V}{c^2}L\right), 0, 0, \gamma(V)qE\left(L - Vt_0\right)\right\},$$

and with:

$$W_{fA}^{\mu} = L_{\nu}^{\mu}(V)W_f^{\nu} = \left\{-c\gamma(V)\mu M_Z g t_0, 0, 0, \mu\gamma(V)M_Z gVt_0\right\},$$

one obtains that force f has a non null four component in S_A, with 'force displacement product' $W_{fA} = \mu\gamma(V)M_Z gVt_0$ [33] and that forces F and f are neither applied simultaneously nor during the same time interval en S_A [34]; and with:

$$Q_A^{\mu} = L_{\nu}^{\mu}(V)Q^{\nu} = \left\{c\gamma(V)(Qc^{-2})V, 0, 0, \gamma(V)Q\right\},$$

one obtains that thermal radiation in S_A has linear momentum $p_{pA} = \gamma(V)M_T V$, according to the Principle of Inertia of Energy (Einstein Equation) [8]. With the relativistic first law of thermodynamics expressed in S_A as:

$$U_{fA}^{\mu} - U_{iA}^{\mu} = W_A^{\mu} + Q_A^{\mu}, \tag{16}$$

where subindex A means the 4-vector as measured in S_A, the corresponding 4-vectors $U_{fA}^{\mu}, U_{iA}^{\mu}, W_A^{\mu}$, and Q_A^{μ} provide a description of the process under consideration, which is equivalent to the description given in S_∞: When equations are fulfilled in S_∞ the corresponding equations in S_A are fulfilled too. For instance, using Eqs. (14-15), in S_A one has:

$$\gamma(V)\gamma(v_f)M_Z\left(v_f - V\right) - \gamma(V)M_Z V = \gamma(V)q\varepsilon\left(t_0 - \frac{V}{c^2}L\right) - \gamma(V)\mu M_Z g t_0 + \gamma(V)\frac{Nh\nu}{c^2}V, \tag{17}$$

$$\gamma(V)\gamma(v_f)M_Z\left(1 - \frac{v_f V}{c^2}\right)c^2 - \gamma(V)M_Z c^2 = \gamma(V)q\varepsilon\left(L - Vt_0\right) + \gamma(V)\mu M_Z gVt_0 - \gamma(V)Nh\nu. \tag{18}$$

For instance, the Impulse-Momentum equation in frame S_A Eq. (17) can be expressed as the sum:

$$\gamma(V)\left[\gamma(v_f)M_Z v_f = \left(q\varepsilon - \mu M_Z g\right)t_0\right] \tag{19}$$

$$+$$

$$-\gamma(V)\frac{V}{c^2}\left[\gamma(v_f)M_Z c^2 - M_Z c^2 = q\varepsilon L - Nh\nu\right], \tag{20}$$

where Eq. (19) is the Impulse-Linear Momentum equation in S_∞ and Eq. (20) is the Energy Equation in S_∞. Because these equations are fulfilled in S_∞, Eq. (17) is fulfilled in S_A. A similar result is obtained for the Energy Equation Eq. (18) in frame S_A.

III. CONCLUSIONS

From Eqs. (9, 10, 11), in the limit $c \to \infty$ [with $\lim_{c\to\infty} \gamma(v)v \to v$ and $\lim_{c\to\infty}\left[\gamma(v) - 1\right]c^2 \to v^2/2$], the corresponding classical description for the considered process ($M_Z \equiv m$) is:

$$mv_f - mv_i = \left(q\varepsilon - \mu mg\right)t_0; \tag{21}$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = q\varepsilon L + Q; \tag{22}$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \left(q\varepsilon - \mu mg\right)L; \tag{23}$$

$$Q = -\mu mgL. \tag{24}$$

Eq. (21) (impulse-momentum equation) is the Newton's second law applied to Z; Eq. (22) is the corresponding first law of thermodynamics ([1], p. 194):

$$\Delta K_{cm} + \Delta U = W + Q,$$

applied to the process [35, 36], with $\Delta U = 0$ because Z temperature is constant – only the conservative force $F = q\mathcal{E}$ applied to Z does work –; Eq. (23) is the centre-of-mass equation which can be obtained considering that ([37], p. 1063):

$$\frac{d[v^2/2]}{d[v]} = v = \frac{dx}{dt};$$

with constant mass m and force F and from Eq. (21) one obtains that:

$$m\Delta[v^2/2] = F\Delta x,$$

for a finite process ([5], pp. 359, 361); Eq. (23) is not a work-energy theorem ([30], p. 506); and Eq. (24), obtained by comparison between Eq. (22) (energy equation) and Eq. (23) (centre-of-mass equation), completes the energy balance for the process, showing that the interchanged thermal energy is the difference (mechanical energy dissipated) between the work done on Z and Z increment in kinetic energy. An equivalent classical description in S_A, from Eqs. (17, 18) in the approximation $c \to \infty$, can be obtained.

Although the 4-vector relativistic thermodynamics first law formalism seems more complicated than the classical Newton's second law and thermodynamics first law approach, it presents some pedagogical advantages: (i) The definition of the 4-vector energy function U^{μ}– the corresponding relativistic internal energy [38]– must be provided, based on the Einstein's equation interpretation for an extended body; (ii) For a conservative force F, its 4-vector work W_F^{μ} can be obtained from the interaction of the body with a work reservoir; (iii) For a non-conservative force f [39] its 'force-displacement product' is zero (it does not do work) and its 4-vector has a null temporal component; (iv) Heat can be characterised in frame S_∞ as a 4-vector Q^μ associated to an ensemble of photons with null total momentum [40]; (v) In frame S_∞ equations, expressed with 4-vectors, are Lorentz invariant and the description of a process in frame S_A is obtained using the Lorentz transformation. Writing a physics law in Minkowski's 4-vector notation, integrating space and time, highlights its invariance, simplifies the formalism and favours calculations. (vi) The relativistic impulse-momentum equation (Newton's second law) and the relativistic energy equation (thermodynamics first law) are simultaneously applied to the process under consideration using the relativistic thermodynamics first law; (vii) The centre-of-mass equation is obtained integrating the corresponding impulse-momentum equation, showing that it is not an energy equation. (viii) Classical physics exercises can be solved using a fully Lorentz covariant formalism that includes mechanics, thermodynamics and electromagnetism 4-vector magnitudes.

APPENDIX

1. *Energy function*. Body Z is a solid crystal composed by N_A atoms of the atomic element ${}^Z_N A$. Body Z energy function $U(T)$ at absolute temperature T is given by [8]:

$$\begin{aligned} U(T) &= N_A u_0 - \tilde{U} + \Delta U(T); \\ u_0 &= N\left(m_\mathrm{p} + m_\mathrm{e}\right)c^2 + \left(Z-N\right)m_\mathrm{n}c^2, \\ \tilde{U} &= \left|\tilde{U}_\mathrm{N}\right| + \left|\tilde{U}_\mathrm{A}\right| + \left|\tilde{U}_\mathrm{C}\right|, \\ \Delta U(T) &\approx N_A \int_0^T c_P(T)\ \mathrm{d}T \end{aligned}$$

where m_p, m_e and m_n are proton, neutron and electron mass, respectively, and $\tilde{U}$ is (negative) body Z formation energy, or energy released when the body is formed from its elementary particles, with $\tilde{U}_N$ as energy nucleus formation (defect of mass), $\tilde{U}_A$ atom energy formation and $\tilde{U}_C$ crystal energy formation (zero-point energy is assumed to be null [41]), respectively ([42], pp. 489-491), and where c_P is solid capacity at constant pressure per atom (assuming expansion coefficient $\alpha = 0$).

The Inertia of Energy Principle [43] allows us to obtain the body's inertia $M(T)$, at temperature T, as [44]:

$$M(T) = U(T)c^{-2}.$$

This equation (the Einstein Eq. [45]) relates two concepts, function energy and inertia, classically apart. An energy function increasing on a body correspondingly increases the body's inertia: A block at a high temperature has more inertia than the same block at a lower temperature. Assuming an isothermal process for body Z during the process, the body's inertia –although temperature dependent – remains constant, with $M(T) = M_Z$.

2. *Solid body in Relativity*. In the Special Theory of Relativity any perturbation on a body constituent travels with finite velocity. No body can be perfectly rigid in Relativity and it deforms under the action of a force applied to it ([46], p. 103). A deformation effect is transmitted to the rest of the body with finite velocity, v s, the velocity of sound in the body's material.

Let b be a characteristic linear dimension of a rigid body Z. If the time interval $\delta t = b/v_s$ –time delayed for a sound wave to travel along the body – is orders of magnitude smaller than interval of time Δt during which forces are applied to body Z, then, and under the action of a moderate force, it can be considered that Z behaves as a rigid solid even from a relativistic point of view. We will consider a robust enough block so that its plastic distortion response to the applied force can be considered negligible.

The expression for a rigid body magnitude –linear momentum, total energy, kinetic energy, etc.– is obtained by changing the mass m in the corresponding expression for a point particle for the inertia M of the body (Principle of Similitude [8]). For instance, for a rigid body moving with velocity v its kinetic energy is $k = [\gamma(v)-1]Mc^2$, which is obtained from the kinetic energy expression $k = [\gamma(v) - 1]mc^2$ for a point particle with mass m.

3. *Work*. Body Z (instantaneous) velocity $\vec{v} = (v, 0, 0)$ in S_∞ is characterized by the 4-vector velocity v^μ, given by [47]:

$$v^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \gamma(v)\left\{v, 0, 0, c\right\},$$

where $\mathrm{d}x^\mu = \left\{\mathrm{d}x, 0, 0, c\ \mathrm{d}t\right\}$ is the 4-vector infinitesimal displacement and τ is body Z proper time, with $\mathrm{d}\tau/\mathrm{d}t = \gamma^{-1}(v)$, where $\mathrm{d}t$ is the corresponding time interval measured in frame S_∞. The 4-vector Minkowski force F^μ is given by Eq. ([6], p. 280)

$$F^\mu = \left\{\gamma(v)q\varepsilon, 0, 0, \gamma(v)c^{-1}q\varepsilon v\right\},$$

and its corresponding infinitesimal 4-vector work δW_F^μ is [45]:

$$\delta W_F^\mu = \left\{cq\varepsilon \mathrm{d}t, 0, 0, q\varepsilon \mathrm{d}x\right\},$$

The 4-vector Minkowski force F^μ is obtained by deriving the 4-vector work δW_F^μ with respect to Z proper time $\mathrm{d}\tau$:

$$F^\mu = c^{-1}\frac{\delta W_F^\mu}{\mathrm{d}\tau},$$

This obtention of F^μ shows that δW_F^μ is itself a 4-vector. For a finite process (with constant F):

$$W_F^\mu = \left\{cq\varepsilon t_0, 0, 0, q\varepsilon L\right\}.$$

In contact with B_∞, the friction force f does not do work, its velocity is null, its 4-vector f^μ is given by:

$$f^{\mu} = \{-\mu M_Z g, 0, 0, 0\},$$

and its 4-vector work W_f^{μ} is:

$$W_f^{\mu} = \{-\mu M_Z g t_0, 0, 0, 0\}.$$

4. *Thermal energy*. A photon j with frequency ν has energy $e_j = h\nu$, linear momentum $p_j = (h\nu/c)u_j$ (one-dimensional), where u_j is its direction $\pm x$ and 4-vector energy function:

$$u_p^{\mu} = \left\{\pm c\frac{h\nu}{c}, 0, 0, h\nu\right\}.$$

For an ensemble of N photons which move chaotically in $\pm x$ direction in a thermal reservoir, its total linear momentum is null [48], $p_p = \sum_j (h\nu/c)u_j = 0$, its total energy is $E_p = \sum_j h\nu|u_j| = Nh\nu$ and its 4-vector energy function is [45]:

$$U_p^{\mu} = \{0, 0, 0, Nh\nu\}.$$

REFERENCES

[1] Serway, R. A., Jewett, J. W. Jr., *Principles pf Physics. A Calculus-Based Text*, 3rd Ed. (Harcourt College Publishers, Orlando, 2002).

[2] Halladay, D., Resnick, R., Walker, J., *Fundamentals of Physics*, 6th Ed. (John Wiley and Sons, New York, 2001).

[3] Knudsen, J. M., Hjorth, P. G., *Elements of Newtonian Mechanics. Including Nonlinear Dynamics*, (Springer, Heidelberg, 2000).

[4] Galili, I., Kaplan, D., *Extending the application of the relativity principle: Some pedagogical advantages*, Am. J. Phys. **65**, 328-335 (1997).

[5] Mallinckrodt, A. J., Leff, H. S., *All about work*, Am. J. Phys. **60**, 356-365 (1992).

[6] Freund, J., *Special Relativity for Beginners. A Textbook for Undergraduates*, (World Scientific, Singapore, 2008).

[7] Güémez, J., *¿Es posible una Termodinámica Relativista?*, Rev. Esp. Fis. **24**, 47-57 (2010).

[8] Güémez, J., *An undergraduate exercise in the first law of relativistic thermodynamics*, Eur. J. Phys. **3**, 1209-1232 (2010).

[9] Hecht, E., *Einstein never approved of relativistic mass*, Phys. Teach. **47**, 336-341 (2009); Hecht, E., *Einstein on mass and energy*, Am. J. Phys. **77**, 799-806 (2009); Hecht, E., *How Einstein confirmes $E_0 = mc^2$*, Am. J. Phys. **79**, 591-600 (2011).

[10] Huggins, E., *Note on Magnetism and Simultaneity*, Phys. Teach. **47**, 587-589 (2009).

[11] Huggins, E., *Special relativity in week one: 1) The Principle of Relativity*, Phys. Teach. **49**, 148-151 (2011); Huggins, E., *Special relativity in week one: 2) All clocks run slow*, Phys. Teach. **49**, 220-221 (2011); Huggins, E., *Special relativity in week one: 3) Introducing the Lorentz contraction*, Phys. Teach. **49**, 148-151 (2011).

[12] Hecht, E., *On defining mass*, Phys. Teach. **49**, 40-44 (2011).

[13] Baierlein, R., *Does nature convert mass into energy?*, Am. J. Phys. **75**, 320-325 (2007).

[14] Battino, R., *"Mysteries" of the First and Second Laws of Thermodynamics*, J. Chem. Educ. **84**, 753-755 (2007).

[15] Barrow, G. M., *Thermodynamics should be built on energy-not on heat and work*, J. Chem. Educ. **65**, 122-125 (1988).

[16] Swartz, C., *Back-of-the-Envelope Physics*, (Johns Hopkins University Press, Baltimore, 2003).

[17] Ringlein, J., Robbins, M. O., *Understanding and illustrating the atomic origins of friction*, Am. J. Phys. **72**, 884-891 (2004).

[18] Besson, U., *Work and energy in the presence of friction: the need for a mesoscopic analysis*, Eur. J. Phys. **22**, 613-622 (2001).

[19] Hilborn, R. C., *Let's ban work from physics*, Phys. Teach. **38**, 447 (2000).

[20] Leff, H. S., Mallinckrodt, A. J., *Stopping objects with zero external work: Mechanics meets thermodynamics*, Am. J. Phys. **61**, 121-127 (1993).

[21] Cavalleri, G., Salgarelli, G., *Revision of the relativistic dynamics with variable rest mass and application to relativistic thermodynamics*, Nuovo Cimento **42**, 722-754 (1969).

[22] Gamba, A., *Physical Quantities in Different Reference Systems According to Relativity*, Am. J. Phys. **35**, 83-89 (1967).

[23] van Kampen, N. G., *Relativistic Thermodynamics*, J. Phys. Soc. Japan **26** Supplement 316-321 (1969).

[24] Romer, R. H., *Heat is not a noun*, Am. J. Phys. **69**, 107-109 (2001).

[25] D Shanks, Monochromatic Approximation of Blackbody Radiation, Am. J. Phys. 24 244-246 (1956).

[26] Huggins, E., *Weighing photons using bathroom scales: A thought experiment*, Phys. Teach. **48**, 287-288 (2009).

[27] Ruby, L., *Teaching Special relativity without calculus*, Phys. Teach. **47**, 231-232 (2009).

[28] Mungan, C. E., *Thermodynamics of a Block Sliding Across a Frictional Surface*, Phys. Teach. **45**, 288-291 (2007).

[29] Tsamparlis, M., *Special Relativity. An Introduction with 200 Problems and Solutions*, (Springer, Berlin, 2010).

[30] Arons, A. B., *Developing the energy concepts in introductory physics*, Phys. Teach. **27**, 506-517 (1989).

[31] Rindler, W., *Relativity. Special, General, and Cosmological*, 2nd Ed. (Oxford University Press, New York, 2006).

[32] Rohrlich, F., *True and apparent transformations, classical electrons and relativistic thermodynamics*, Nuovo Cimento **45**, 76-83 (1966).

[33] Mungan, C. E., *The Bernoulli equation in a moving reference frame*, Eur. J. Phys. **32**, 517-520 (2011).

[34] Adler, C. L., *Magnetism and simultaneity*, Phys. Teach. **47**, 221-223 (2009).

[35] Erlichson, H., *Internal energy in the first law of thermodynamics*, Am. J. Phys. **52**, 623-625 (1984).

[36] Jewett, J. W. Jr., *Energy and the Confused Student IV: A global approach to energy*, Phys. Teach. **46**, 38-43

(2008).

[37] Arons, A. B., *Development of energy concepts in introductory physics courses*, Am. J. Phys. **67**, 1063-1067 (1999).

[38] Lehrman, R. L., *Energy Is Not The Ability To Do Work*, Phys. Teach. **11**, 15-18 (1973).

[39] Keeports, D., *The common forces: Conservative or nonconservative?*, Phys. Educ. **41**, 219-222 (2006).

[40] Kolbenstvedt, H., *The mass of a gas of massless photons*, Am. J. Phys. **63**, 44-46 (1995).

[41] Mungan, C. E., *Chemical potential of the one-dimensional simple harmonic oscillators*, Eur. J. Phys. **30**, 1131-1136 (2009).

[42] Hecht, E., *An Historico-Critical Account of Potential Energy: Is PE Really Real?*, Phys. Teach. **41**, 486-493 (2003).

[43] Hecht, E., *There Is No Really Good Definition of Mass*, Phys. Teach. **44**, 40-45 (2006).

[44] Hecht, E., *Energy and Change*, Phys. Teach. **45**, 88-92 (2007).

[45] Güémez, J., *Termodinámica Relativista: Una aproximación didáctica al Primer Principio*, Lat. Am. J. Phys. Educ. **5**, 72-91 (2011).

[46] Green, D., *The strange world of classical physics*, Phys. Teach. **48**, 101-105 (2010).

[47] Brehme, R. W., *The Advantage of Teaching Relativity with Four-Vectors*, Am. J. Phys. **36**, 896-901 (1968).

[48] Gabovich, A. M., Gabovich, N. A., *How to explain the non-zero mass of electromagnetic radiation consisting of zero-mass photons*, Eur. J. Phys. **28**, 649-655 (2007).