Understanding the measurement theory in quantum mechanics

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Abstract

In this paper we show the measurement process and understanding the measurement theory. We review the Copenhagen interpretation and Von Neumann's theory and Zurek's theory of measurement. We try to show correspondence between measurement process and different interpretations. My motivation of this paper is to enable beginning students to start exploring the vast literature on this subject.

Keywords: Measurement theory, Copenhagen interpretation, Von Neumann's theory, Zurek's theory.

Resumen

En este trabajo se muestra el proceso de medición y comprensión de la teoría de la medición. Revisamos la interpretación de Copenhague y la teoría de Von Neumann y la teoría de Zurek de medición. Tratamos de mostrar la correspondencia entre el proceso de medición y las diferentes interpretaciones. Mi motivación de este trabajo es que los estudiantes comiencen a explorar la vasta literatura sobre este tema.

Palabras clave: Teoría de la medición, interpretación de Copenhague, teoría de Von Neumann, teoría de Zurek.

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I. INTRODUCTION

In quantum theory, measurement is used to introduce probabilities into the theory, and this feature is the source of many conceptual difficulties and paradoxes. In particular, it gives the misleading impression that one cannot apply statistical ideas to quantum processes in the absence of measuring devices, *e.g.*, to the decay of unstable particles in the center of the Sun, or interstellar space. In other words, a measurement of some observable quantity of the system would cause the system to fall into a specific quantum mechanical state (the infamous reduction or collapse of the wave function). Wave function collapse (or reduction) was introduced by Von Neumann [1] as a separate mode of time evolution for a quantum system, quite distinct from the unitary time evolution implied by Schrödinger's equation. The concept leads to a number of conceptual difficulties, and is one of the sources of the widespread (but incorrect) notion that there are superluminal influences in the quantum world. From the consistent histories perspective, wave function collapse is a mathematical procedure for calculating certain kinds of conditional probabilities that can be calculated by alternative methods, and thus has nothing to do with any physical process. This idea in itself presents a tricky problem to the physicist. If it is hoped that quantum mechanics is the theory of the Universe then how can there be an external observer to make measurements of the Universe? The requirement of the existence of an external observer means that no system can be closed. This leads to significant philosophical problems in considering the Universe as a supposedly closed system.

Before some observable of the system is measured, what state is the system in? Classically, we might insist that the system had to occupy some particular state. But in quantum mechanics, the system exists in a fuzzy mix of several allowed states, a quantum superposition. Dirac is credited with the observation that the principle of superposition is one of the two most important pillars upon which quantum theory is built (the other being the Schrödinger equation) [2]. Once a measurement is made of the system it is forced into one particulate state. The measurement destroys the probabilities and forces one particular out come.

II. THE MEASUREMENT PROCESS

We now consider a highly idealized selective measurement of an observable. Suppose we consider a very tiny detector so that if the detector is designed to measure some observable *O*, it will leave the measured object, at least for an instant, in a zero-uncertainty state $\Delta O = 0$ of that observable, *i.e.*, in an eigenstate of the operator *O*, and record the eigenvalue found. Suppose, once again, that the object is an electron, and the detector is a Stern-Gerlach apparatus which measures the electron spin in the z-direction. A red light goes on if the electron is spin-up, a green light goes on if it is spin down. The electron exist the apparatus, which is enclosed in a black box, in an eigenstate of s^z. In order to explain how a detector works, one would begin by analyzing the interaction between the detector and the particle, and describe the evolution of the particle-detector system in terms of some Hamiltonian, involving the degrees of freedom of both the particle and the detector. The detector is designed to measure s^z. This means that if the initial state of the particle-detector system is

$$|\psi_0\rangle = |up\rangle|ready\rangle. \tag{1}$$

Where $|up\rangle$ means the electron is in a spin-up state, and $|ready\rangle$ means that the detector has been switched on, then the interaction between the electron and detector leaves the combined system in the final state

$$|\psi_f\rangle = |up\rangle|red\rangle. \tag{2}$$

Where $|red\rangle$ indicates that red light on the detector is on. It is assumed that transition from the initial to final state is adequately explained by the Schrödinger equation, and thus we write, schematically,

$$|up\rangle|ready\rangle \longrightarrow |up\rangle|red\rangle. \tag{3}$$

Similarly, if the particle starts in a spin-down states,

$$|down\rangle|ready\rangle \longrightarrow |down\rangle|green\rangle. \tag{4}$$

We know The Schrödinger equation is a linear equation. This means that if $\psi_a(x,t)$ and $\psi_b(x,t)$ are both solutions of the Schrödinger equation, and if the initial state is $\psi_0 = A\psi_a(x,0) + B\psi_b(x,0)$ then the corresponding final state (at time *t* = *T*, say) is $\psi_f = A\psi_a(x,T) + B\psi_b(x,T)$ suppose, then, that the electron entering the detector is in the spin state $|\psi\rangle = A|up\rangle + B|down\rangle$ so that the initial particle –detector state is

$$|\psi_0\rangle = |\psi\rangle|ready\rangle = A|up\rangle|ready\rangle + B|down\rangle|ready\rangle. \tag{5}$$

Then, from the linearity of the Schrödinger equation, we can conclude that the particle-detector system must end up in the entangled final state

$$|\psi\rangle = A|up\rangle|red\rangle + B|down\rangle|green\rangle. \tag{6}$$

When we add a human observer to the system, the quantum-mechanical assault on common-sense only gets worse. Denote the initial state of the observer by ket $|waiting\rangle$. When the observer sees a red light flash, her state changes to a new state, denoted $|I-saw-red\rangle$, likewise, if the green light flashes, the observer is left in the state $|I-saw-green\rangle$. The Hamiltonian describing the interaction between particle, detector, and observer is assumed to lead to

$$|up\rangle|ready\rangle|waiting\rangle \longrightarrow |up\rangle|red\rangle|I-saw-red\rangle. \tag{7}$$

Similarly, if the particle starts in a spin-down state,

$$|down\rangle|ready\rangle|waiting\rangle \longrightarrow |down\rangle|green\rangle|I-saw-green\rangle. \tag{8}$$

Then if the instead the particle starts off in the superposition of spin states $|\psi\rangle$ of $|\psi\rangle = A|up\rangle + B|down\rangle$ so the initial state of the combined system is

$$|\psi_0\rangle = |\psi\rangle|ready\rangle|waiting\rangle = A|up\rangle|ready\rangle|waiting\rangle + B|down\rangle|ready\rangle|waiting\rangle. \tag{9}$$

Then by the linearity of the Schrödinger equation, we are forced to conclude that the particle-detector-observer system ends up in the entangled final state

$$|\psi_f\rangle = A|up\rangle|red\rangle|I-saw-red\rangle + B|down\rangle |green\rangle|I-saw-green\rangle. \tag{10}$$

The observer has also ended up in a quantum superposition, in which she is neither in the $|I-saw-red\rangle$ state, nor in the $|I-saw-down\rangle$ state. And this strange conclusion is deduced simply from the assumption that a particle in a spin up (down) state always leads the observer to see red (green) state, and from the linearity of the Schrödinger equation.

III. THE COPENHAGEN INTERPRETATION

The Copenhagen interpretation was the philosophical description of quantum mechanics put forward by the group of scientists working at the institute for Atomic Studies in Copenhagen –heading the group was Niels Bohr. One can summarize it as follows:

(1) A measurement is an interaction between a (measured) quantum system S_q, obeying the rules of quantum mechanics, and a (measuring) classical system A_c, obeying the rules of classical mechanics. This A_c induced the reduction of the state vector of S_q thereby preventing us

from observing the rich linear structure of the whole Hilbert space H_{Sq}.

(2) The measurement process by itself, i.e., the interaction between S_q and A_c, cannot be investigated by quantum mechanics. It may be considered as instantaneous, or as occurring within a time interval that is too short to be investigated. (See to the equation of (5)) .the reduction of the state vector of S_q should thus be viewed as an instantaneous, discontinuous process.

(3) Quantum mechanics is the ultimate scientific theory of Nature. From (2) and (3) we deduce that the measurement process cannot be investigated by Science. The claim of (1) is that a measurement is an interaction between the quantum world $Q = S_q$ (same to electron in section 2) and a subset A_c of the classical world C (same to classical detector in section 2), while the remainder E_c of C (for example an human) does not matter. A measurement thus implies a fundamental division of the Universe U between Q and $C = A_c \cup E_c$. This division may be written as

$$U = Q \cup C = S_q \cup (A_c \cup E_c).$$

Where $Q \cap C = \emptyset$, (the state vector of U is similar to equation of 9). The boundary between Q and C that divides U must be adjusted to each particular situation: first we define Q, and then we define C as U – Q.

The division of U may be viewed as arbitrary and inherent not to U but to our way of probing U through quantum mechanics. It thus triggered an intense debate among scientists and philosophers. This debate is presented by the opposition between Bohr, the chief of the opponents of the Copenhagen interpretation, and Einstein who wanted to deny, *i.e.*, to show that quantum mechanics is only a provisional theory of Nature.

The Copenhagen interpretation was very successful at describing laboratory systems that contain a classical observer equipped with a measuring apparatus and the system on which the measurement was done (*e.g.*, the Stern – Gerlach experiment, etc). So, to give a description of the system the world had to be split somehow into a classical part and a quantum mechanical part, a very unsatisfactory idea , in particular , because in some cases it is not clear at all where to make the cut. Also it follows that the system under measurement has to be necessarily open, to interact with the observer. These two characteristics of the Copenhagen interpretation make it inapplicable to the Universe as a whole, because the Universe is a closed system and obviously there is no observer external to it. Another, possibly undesirable, characteristic is that the concept of a classical trajectory disappears somewhere between the very small and the everyday world in which we live. Not only does the Copenhagen interpretation suggest that classical trajectories cannot exist but also that very small objects do not have to be in any particular place until they are observed. Classically, one tends to think of objects being in very – well – defined positions! Two physical quantities P and Q of S whose associate operators P and Q do not commute are incompatible in the sense that the knowledge by measurement of one of these quantities precludes that of the other, at the same time.

V. BOHR'S COMPLEMENTARITY

Theorem: Two physical quantities P and Q of S whose associate operators $\hat{P}$ and $\hat{Q}$ do not commute are incompatible in the sense that the knowledge by measurement of one of these quantities precludes that of the other, at the same time.

Bohr's interpretation of theorem is called the complementarity principle. It consists in noting that the two physical quantities mentioned in the theorem cannot be measured by the same apparatus. So the reason for the incompatibility of P and Q comes from the incompatibility of the apparatus A_c^p and A_c^Q that are needed to measure them. Bohr concluded that A_c^p and A_c^Q, or equivalently P and Q, are complementary for our understanding of S.

Einstein's interpretation of the above theorem was quite different. He indeed used this theorem (in EPR paradox - 1935) [3] to show that quantum mechanics does not give a complete description of the physical reality of S.

V. VON NEUMANN'S THEORY OF MEASUREMENT

Von Neumann developed a theory of measurement that is similar to that of Copenhagen interpretation, except that he considered the measuring apparatus as obeying the rules of quantum mechanics like the measured system. We can summarize his theory as follows:

(1) A measurement is an interaction between a (measured) quantum system S_q and a (measuring) quantum system A_q, or apparatus, which both obey the rules of quantum mechanics. This is A_q, which induces the reduction of the state vector of S_q, mentioned before thereby preventing us from observing the linear structure of the whole H_{Sq}.

(2) The measurement process by itself, i.e., the interaction between S_q and A_q, is a process in two stages. The first stage consists in a unitary time evolution of state vector of the combined system $S_q \cup A_q$, leading to a pure correlation between state vectors of S_q and A_q. The second stage cannot be investigated by quantum mechanics, it may be considered as an instantaneous, discontinuous process.

(3) Quantum mechanics is the ultimate scientific theory of Nature.

From (2) and (3) we deduce that only the second stage of the measurement process cannot be investigated by Science. The claim of (1) is that a measurement is an interaction between the two subsets of the quantum world $Q = S_q \cup A_q$, (S_q assumed that is an electron and A_q is the quantum detector). The classical world $C = E_c$ does not matter. A measurement thus implies a fundamental division of the Universe U between Q and C. This division may be written as

$$U = Q \cup C = (S_q \cup A_q) \cup E_c.$$

Where $Q \cap C = \emptyset$. The boundary between Q and C that divides U must be adjusted to each particular situation: First we define Q, and then we define C as U – Q.

To see the two stages of the measurement process in more detail, let us suppose that Hilbert space H_{Sq} and H_{Aq} associated, respectively, with S_q and A_q are both two dimensional. Associated with the to be measured physical quantity P of S_q is the operator P whose spectrum is given by

$$\hat{P}|1\rangle = P_1|1\rangle,$$
$$\hat{P}|2\rangle = P_2|2\rangle.$$

As we shall see, P should be correlated with a physical quantity A of $A_{q,}$ called the pointer observable, whose associate operator A has a spectrum given by

$$\hat{A}|a_1\rangle = a_1|a_1\rangle,$$
$$\hat{A}|a_2\rangle = a_2|a_2\rangle.$$

Clearly $\{|1\rangle, |2\rangle\}$forms a basis of $H_{Sq,}$ and $\{|a_1\rangle, |a_2\rangle\}$ forms a basis of $H_{Aq,}$ which is called the pointer basis. the Hilbert space $H_{Sq} \otimes H_{Aq}$ associated with $Q = S_q \cup A_q$ is thus the four – dimensional space generated by the basis $\{|i\rangle, |a_j\rangle\}_{i,j=1,2}$.

Let Q be in the initial state

$$|\Phi(t_i)\rangle = |\emptyset(t_i)\rangle \otimes |a(t_i)\rangle = (\alpha_i|1\rangle + \beta_i|2\rangle) \otimes (\mu_i|a_1\rangle + \nu_i|a_2\rangle). \qquad (11)$$

Where

$$\alpha_i = \langle 1|\emptyset(t_i)\rangle,$$
$$\beta_i = \langle 2|\emptyset(t_i)\rangle.$$

Satisfy, $|\alpha_i|^2 + |\beta_i|^2 = 1$ and where

$$\mu_i = \langle a_1|a(t_i)\rangle,$$
$$\nu_i = \langle a_2|a(t_i)\rangle,$$
$$\text{Satisfy } |\mu_i|^2 + |\nu_i|^2 = 1.$$

The first stage of the measurement process consists in the unitary time evolution from (11) to the pure, correlated state vector

$$|\Phi(t_c)\rangle \equiv \widehat{U}(t_c, t_i)\rangle|\Phi(t_i)\rangle = \alpha_i|1\rangle \otimes |a_1\rangle + \beta_i|2\rangle \otimes |a_2\rangle. \qquad (12)$$

In (1.8), there is a pure correlation between the basis vectors of H_{Sq} and the pointer basis vectors: $|1\rangle$ is correlated with$|a_1\rangle$ and $|2\rangle$ is correlated with $|a_2\rangle$.

The second stage of the measurement process is the postulated, instantaneous reduction of (12), whose associate density matrix is

$$\hat{\rho}_c \equiv |\Phi(t_c)\rangle\langle\Phi(t_c)| = |\alpha_i|^2|1\rangle\langle 1| \otimes |a_1\rangle\langle a_1| +$$
$$\alpha i \beta i * 12 \otimes a1a2$$
$$+\alpha_i^* \beta_i|2\rangle\langle 1| \otimes |a_2\rangle\langle a_1| + |\beta_i|^2|2\rangle\langle 2| \otimes |a_2\rangle\langle a_2|. \qquad (13)$$

To the mixed state vector whose associate density matrix is

$$\hat{\rho}_r \equiv |\alpha_i|^2|1\rangle\langle 1| \otimes |a_1\rangle\langle a_1| + |\beta_i|^2|2\rangle\langle 2| \otimes |a_2\rangle\langle a_2|. \qquad (14)$$

The reduced density matrix (14), thus obtained by cancelling the off-diagonal terms of the correlated density matrix (13), has the following physical meaning: After a measurement of P, S_q can be either in the eigenstate $|1\rangle$ (with a probability equal to $|\alpha_i|^2$) or in the eigenstate $|2\rangle$ (with a probability equal to $|\beta_i|^2 = 1 - |\alpha_i|^2$). This clearly expresses the reduction of the state vector $|\Phi(t_i)\rangle$ of S_q.

From (14), we see that the ban on S_q from being in a superposition of $|1\rangle$ and $|2\rangle$ after a measurement of P is equivalent to the ban on A_q from being in a superposition of the pointer basis vectors $|a_1\rangle$ and $|a_2\rangle$ after a measurement of P. the reason for this ban remains mysterious because the reduction (13)$\longrightarrow$(14), which is necessary to produce it, is a postulate.

The simplest interpretation was given by Von Neumann theory, who urges us to follow the chain of events into the brain of the observer. The quantum detector is in a superposition of red light/green light states, and it emits photons in a superposition of the corresponding frequencies. The photons reach the retina of the observer, and certain neurons are left in a superposition of excited/un-excited states. The message from the retina travels to the cerebral cortex, very large collections of neurons are now in quantum superpositions, and the brain remains in such a superposition until, at some point, a sensation occurs. at the instant of conscious perception , the observer (E_c), the quantum detector (A_q) and even the particle (S_q), jump into one or the other of the up/down , red/green states.

What this view is suggesting is that human consciousness causes the wave function to collapse, with a certain probability, into one or another of the possible observer, that the mental function described as awareness or consciousness cannot be described by the Schrödinger equation, it is not a physical process in the usual sense ((13)$\longrightarrow$(14)).

VI. ZUREK'S THEORY OF MEASUREMENT

Zurek [4, 5] developed a new theory of measurement that we can summarize his theory as follows:

(1) A measurement is an interaction between a (measured) quantum system S_q and a (measuring) quantum system A_q, or apparatus, and the remainder E_q of the Universe U. Each of these three systems is assumed to obey

the rules of quantum mechanics. This is the environment E_q of $S_q \cup A_q$, which forces, through the so called environment induced superselection rules, A_q to induce the reduction of the state vector of S_q thereby preventing us from observing the rich linear structure of the whole H_{Sq}.

(2) The measurement process by itself, *i.e.*, the interaction between S_q, A_q, and E_q is a process in two stages. The first stage consist in a unitary time evolution of the state vector of the Universe $U = S_q \cup A_q \cup E_q$, leading to a pure correlation between state vectors of S_q and A_q. the second stage consist in a unitary time evolution of the state of U, transforming the pure correlation between state vectors of S_q and state vectors A_q into a pure correlation between state vectors of S_q and state vectors of A_q. The latter stage may be viewed as equivalent to the reduction of the state vectors of S_q induced by A_q, but it is a continuous, (observable) process occurring within a finite time interval. (We can observe gradually the loss of the rich linear structure of the whole H_{Sq}, associated with the progressive reduction of the vector of S_q.

(3) Quantum mechanics is the ultimate scientific theory of Nature.

From (2) and (3) we deduce that the whole measurement process can be investigated by science. The claim of (1) is that a measurement is an interaction between the three subsets of the Universes $U = S_q \cup A_q \cup E_q$. the whole Universe is assumed to be quantum, there is no classical world. In particular, there is no longer the arbitrary division between the quantum world and the classical world that must be adjusted to each particular situation in Copenhagen interpretation's and Von Neumann's theories.

VII. CONCLUSION AND FURTHER COMMENTS

This survey explores various ways of interpret measurement in quantum mechanics. In standard quantum mechanics and some different ways of measuring it. The approach of Zurek lead to the concept of decoherence [6]. We think E_q in Zurek's theory is awareness of observer and also it is infinite dimensional. So can be written the analogue of the Eq. (14).

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