

A physics model for weight loss by dieting



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Abstract

A person's average weight is determined by the equilibrium between his caloric intake (from food and liquids) and his energy loss (primarily by radiation and water evaporation). By dieting, one's weight can decrease significantly over a period as short as a couple dozen weeks. A simple physics-based model is developed that describes such a change in weight with time. The predictions are in reasonable accord with televised results of disciplined dieting plans.

Keywords: Human body, weight change, diet, energy loss.

Resumen

El peso promedio de una persona está determinado por el equilibrio entre su ingesta de calorías (de alimentos y líquidos) y su pérdida de energía (principalmente mediante la radiación y evaporación de agua). Al hacer dieta determinada, el peso propio puede disminuir significativamente en un período tan corto como un par de docenas de semanas. Se desarrolla un modelo simple basado en la física que describe un cambio en el peso con el tiempo. Las predicciones son razonables de acuerdo con los resultados televisados de los planes de dieta disciplinados.

Palabras clave: Cuerpo humano, cambio de peso, dieta, pérdida de energía.

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The weight w of a person varies with time t due to the competition between the average rate a at which he gains weight from the consumption of food calories and the average energetic loss from the surface area S of his body. One can neglect the nondigestible solid part of foods that passes right through the body, and the weight of the water content in food and drink that is lost as sweat and urine. Then the intake counts only the food calories in one's diet, averaged over a time period (say 2 or 3 days) that is long enough to smooth out the variations in when and what a person consumes, but short enough to respond to dieting trends. The energetic loss from the body is principally radiative, plus the latent heat of vaporization of transpired water from one's skin and breath. This loss varies depending on the temperature of one's surroundings, what one is wearing, and how much one exerts oneself and so is again averaged over about 2 or 3 days to smooth out such fluctuations. A person's weight is commonly measured in the United States in pounds, but it properly refers to one's body mass in kilograms.

To estimate the surface area S of a person (whom we will take to be a male adult), we will model him as a right circular cylinder of radius r and height h . His volume is then $V = \pi r^2 h$. Dieting only alters a person's mean radius r and not his height h . Neglecting the differences in density of fat, muscles, organs, and the like, a person's volume V is proportional to his weight w and thus $r \propto \sqrt{w}$. Furthermore, *Lat. Am. J. Phys. Educ. Vol. 6, No. 3, Sept. 2012*

since $h \gg r$ for a typical adult, we can neglect the top and bottom end faces of the cylinder and estimate $S \approx 2\pi rh \propto \sqrt{w}$. This square-root dependence of the surface area on weight agrees with the Mosteller formula, commonly used in the medical community [1].

Putting together the factors described in the previous two paragraphs, we deduce the differential equation

$$\frac{dw}{dt} = a - b\sqrt{w}, \quad (1)$$

where w is in kg, a in kg/s (proportional to one's consumption in Cal/day), and b in $\text{kg}^{1/2}/\text{s}$ (related to radiative emission from the body at 37°C , radiative absorption from the surroundings at about 20°C , and an estimate of the rate of water transpiration from the lungs and skin). Change variables to $w = x^2$ in Eq. (1) and divide it through by $2x$ to get

$$\frac{dx}{dt} = \frac{X^2}{T} \left(\frac{1}{x} - \frac{1}{X} \right), \quad (2)$$

where the two constants a and b have been re-expressed in terms of two new constants $X \equiv a/b$ and $T \equiv 2a/b^2$. Their physical significance is that $W = X^2$ is the person's equilibrium weight and T is a time constant characterizing

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the rate at which one loses weight if $x > X$. Introducing the dimensionless variables $z \equiv x/X$ and $\tau \equiv t/T$, Eq. (2) can be rewritten as

$$\frac{dz}{d\tau} = \frac{1}{z} - 1. \tag{3}$$

This equation can be separated and the indefinite integral evaluated by parts to get

$$\tau = -z \ln(z-1) + \int \ln(z-1) dz. \tag{4}$$

Putting $y \equiv z-1$, the remaining integral becomes $\int \ln y dy$ which can again be integrated by parts to give $y \ln y - y$, so that

$$\tau = -z \ln(z-1) + (z-1) \ln(z-1) - (z-1). \tag{5}$$

Taking the initial value to be $z = z_0$ when $\tau = 0$, we finally obtain the value of the definite integral as

$$\tau = \ln\left(\frac{z_0-1}{z-1}\right) + (z_0 - z). \tag{6}$$

Assuming the person is initially above his target equilibrium weight, then z decreases from z_0 to 1 as time elapses, and both terms on the right-hand side of Eq. (6) are positive. This equation cannot be analytically inverted to find $z(\tau)$. However it can be graphed, from which one can numerically determine the person's weight as a function of time. For example, Fig. 1 plots the case of a person who begins dieting when he is double his target weight, so that $z_0 = \sqrt{2}$.

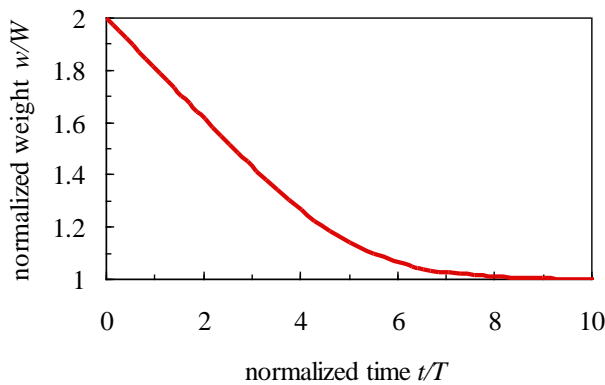
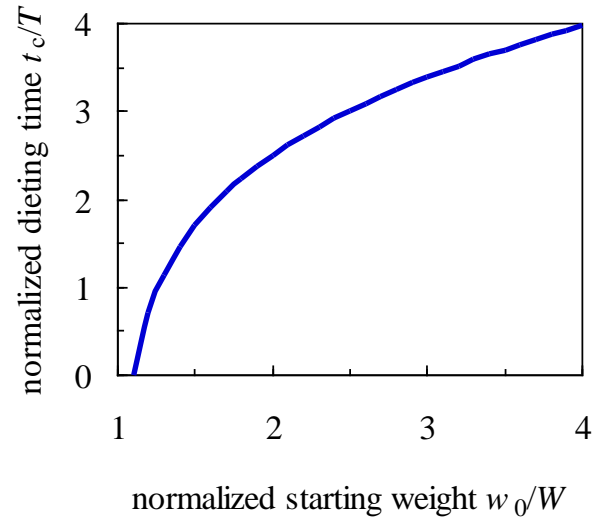


FIGURE 1. Weight w of a person who begins dieting when he is double his new equilibrium weight W .

We see from Fig. 1 that one's target weight is approached asymptotically. To characterize the time required for the diet to succeed, one could compute how long it takes to shed say

90% of the pounds one is trying to lose. That dimensionless time τ_c is given by substituting $z = \sqrt{1.1}$ into Eq. (6) and is plotted in Fig. 2.

FIGURE 2. Time required for a diet to succeed as a function of the fraction of one's initial to one's target weight.



To convert these results into an actual time in weeks, we calibrate against some real numbers as follows. Equation (1) can be rewritten as

$$-\frac{1}{w_0} \frac{dw}{dt} = \frac{2W}{Tw_0} \left[\sqrt{\frac{w}{W}} - 1 \right]. \tag{7}$$

The left-hand side of this equation evaluated at $t=0$ is the initial rate of fractional weight loss. According to the television show "The Biggest Loser" [2], contestants typically lose about 5% of their weight in the first week of their diet. Suppose the person begins at double their target weight. Equation (7) then implies that $T = (\sqrt{2} - 1)/0.05 = 8.3$ weeks and Eq. (6) predicts that it will take him $\tau_c T = 21$ weeks to reach a weight within 10% of his target. Finally, if we assume dieting changes a but not b , then $X \equiv a/b$ implies that $W \propto a^2$. Thus to halve one's weight, one must reduce one's calorie intake by 29%. To give a feel for the numbers in Eq. (1), suppose the person's target weight is 80 kg, starting from 160 kg. Then the defining formulas for X and T given in connection with Eq. (2) imply that the dieting person is gaining weight at a rate of $a \approx 19$ kg/week with his reduced caloric intake, while simultaneously losing weight at a rate of $b\sqrt{w_0} \approx 27$ kg/week when he starts his diet, which falls to a balancing value of $b\sqrt{W} \approx 19$ kg/week when he reaches his target. Note that he is losing 8 kg/week at the beginning of his diet and zero at the end of it, for an average loss of 4

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kg/week. Multiplying that rate by the 21 weeks of his diet correctly predicts that he will lose about 80 kg, in agreement with typical results seen on the television show [2].

REFERENCES

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- [2] <http://www.nbc.com/the-biggest-loser/> (also see the similar Spanish show “Dale Con Ganas” with website at <http://daleconganas.com/>).