Heat transfer through of a homogeneous and isotropic slab excited by a periodic light beam: Simulation by COMSOL Multiphysics

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Abstract

In the study of the heat transfer and their applications, we are interesting in the solution of the heat diffusion equation with a particular boundary conditions kind congruent to the physical circumstances of the problem under consideration. However, the complexity of the mathematical problem proposed to describe the experimental conditions is, many times, very complex and only possible to obtain a mathematical model based in an analytical solution with several approximations that generally not reflect the physical reality of the problem. Fortunately, nowadays the development of the advanced numerical methods and computing systems allow us the application of high level software for obtain an approximate solution to a complex mathematical problem with a boundary conditions that are congruent with the physical reality. For this reason, we solved the heat conduction equation for a homogeneous and isotropic solid slab exited by a periodic light beam on its front surface, with the boundary conditions congruent with the physical situation, by means of application the COMSOL Multiphysics software. In addition, we compare our simulated results with the experimental ones of the temperature difference on the rear surface of the crystalline silicon wafer as a function of the exposure time obtained by means of infrared Photothermal Radiometry. Our results show a significant agree between the experimental and simulated results, which demonstrate the utility of this methodology in the study of the thermal response in solids.

Keywords: Heat transfer, thermal properties, thermal waves, simulation, COMSOL Multiphysics.

Resumen

En el estudio de la transferencia de calor y sus aplicaciones, estamos interesados en la solución de la ecuación de difusión de calor con una clase particular de condiciones de frontera congruentes a las circunstancias físicas del problema en cuestión. Sin embargo, la complejidad del problema matemático propuesto para describir las condiciones experimentales es, muchas veces, muy complejo y solo es posible obtener un modelo matemático basado en una solución analítica con varias aproximaciones que generalmente no reflejan la realidad física del problema. Afortunadamente, hoy día el desarrollo de métodos numéricos avanzados y sistemas de cómputo nos permiten la aplicación de software de alto nivel para obtener una solución aproximada a un problema matemático complejo con condiciones a la frontera congruentes a la realidad física. Por esta razón, en este trabajo resolvemos la ecuación de conducción de calor para un sólido homogéneo e isotrópico en forma de placa excitado en su superficie frontal, con condiciones de frontera congruentes con la situación física, mediante la aplicación del software COMSOL Multiphysics. Además, comparamos nuestros resultados de la simulación con los resultados experimentales obtenidos de la diferencia de temperatura en la superficie trasera de una muestra de silicio cristalino como una función del tiempo de exposición, obtenido mediante radiometría Fototérmica infrarroja. Nuestros resultados muestran una notable concordancia entre los resultados simulados y los experimentales, lo que demuestra la utilidad de esta metodología en el estudio de la respuesta térmica en sólidos.

Palabras clave: Transferencia de calor, propiedades térmicas, ondas térmicas, simulación, COMSOL Multiphysics.

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I. INTRODUCTION

The heat diffusion equation (HDE) is a partial differential equation of fist order in time and second order in the spatial coordinates, therefore, is necessary to specify one condition in time, the initial condition, and two boundary conditions for each coordinate necessary in the description of the system, the boundary conditions of the problem. There are three kinds of boundary conditions generally used in problems of heat transfer. Dirichlet condition, also called first kind boundary condition, corresponds when the temperature surface is known. Neumann condition, or second kind boundary condition, corresponds when the heat flux is known. And Robin condition, also known as third kind boundary condition or newton law of cooling, corresponds to the existence of convection heating (or cooling) at the surface [1]. The choice of which boundary condition will be applied depends of the physical conditions existing at the boundaries of the medium. Of course, these three types of boundary conditions don't describe, nevertheless, all real conditions that occur in practice, such as body heating and cooling by radiation, the melting or freezing of bodies or complex heat transfer. COMSOL Multiphysics is a powerful Finite Element (FEM) Partial Differential Equation (PDE) solution engine [2] useful to obtain a numerical solution in complex problems.

Development and applications of heat transfer is of fundamental importance in many branches of engineering since provides economical and efficient solutions for critical problems encountered in many engineering items of equipment [3, 4].

Photothermal (PT) techniques are a high sensitivity methods used to investigate the thermal properties of a variety of materials. Progress in the PT sciences has been made mainly due to continued improvements in the development of light sources and equipment for data collection and processing [5, 6, 7]. In particular, Photothermal Radiometry (PTR) originally proposed by Nordal and Kanstad [8] is one of the most important techniques because its detection method involves nondestructive, noncontact remote sensing. This technique consists of impinging a modulated light beam onto the sample and the resulting heat flux produces a periodic temperature distribution, called "thermal wave". As a result, infrared (IR) Planck radiation is emitted and subsequently detected by an IR detector. Since temperature rise depends on the thermal properties of the sample, modulated PTR can be used to investigate thermal response of materials [9, 10].

II. EXPERIMENTAL

The PTR system consists of a sample holder fixed to an aluminum rail, the light beam is generated by a Lasever laser, 473-532nm wavelength and 1W power, with integrated driver to receive TTL signals, a lock-in amplifier with two functions: send the TTL signal to synchronize the shutter driver and capture data. The IR signal is detected with an Exergen smart IR t/c sensor, with integrated scanning system for sending data port RS-232 and it was implemented a LabView software for automation and experimental data acquisition.

It was considered a crystalline silicon sample of 400μm thickness. The laser beam impinges on the sample front face in an area of $15mm^2$, which was covered with a thin layer of painting graphite to prevent the laser beam reflection. The sample was heated in an interval of 550 seconds and the temperature rise was measured on the sample rear face. We work with 0.05Hz frequency, in order of obtain the temperature oscillations, which correspond to the frequency assigned in the Lock-in. Fig. 2(a) shows the experimental results of the temperature increase versus time at a frequency of 0.05Hz.

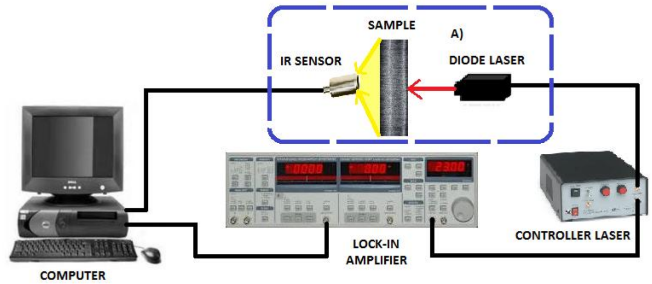


FIGURE 1. Experimental setup of the Infrared Photothermal Radiometry system.

III. SIMULATION

The heat transfer analysis was carried out on the front side of the sample. The temperature profile was obtained at a frequency of the incident laser beam of 0.05Hz, at different times. It was obtained the temperature distribution in the sample as a function of time by means of the solution of the heat diffusion equation with the corresponding boundary conditions:

$$-\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)=\rho c\frac{\partial T}{\partial t}, \tag{1}$$

$$q\big|_{x=0}=q_0\left[1+\cos(2\pi ft+\pi)\right], \tag{2}$$

$$-k_s\Delta T\big|_{x=L}=-k_a\Delta T\big|_{x=L}. \tag{3}$$

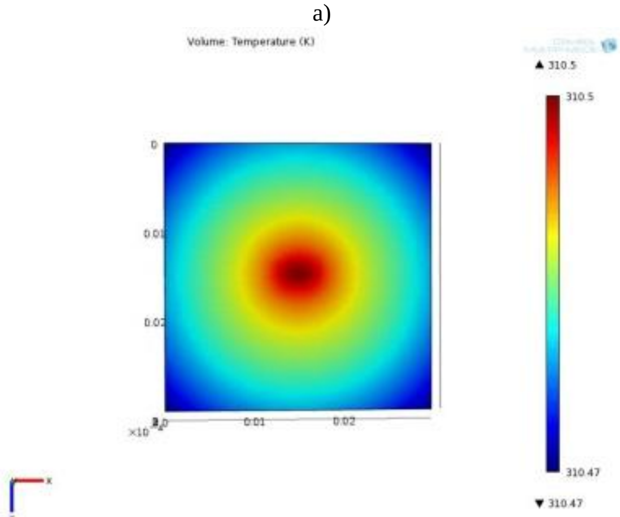
Here, f is the modulation frequency of the incident beam, L is the sample thickness, k_s is the thermal conductivity of the sample, k_a is the thermal conductivity of the air, ρ is the density, c is the specific heat and q_0 is the heat flux.

The models were developed under version 4.1 and then worked under version 4.2. The physics used in COMSOL was heat transfer; the interface provides the equations, boundary conditions and sources for modeling. It was used the default physical model heat transfer in solids. The geometry was designed in COMSOL by using composite objects and blocks. The equations were defined as variables under global definitions. In the heat transfer model we do not use the material library. Instead, all necessary values like effective heat capacity, heat conductivity and specific density were described as parameters in all cases. All simulations were performed with a 3D model and it was used a tetrahedral mesh with a minimum of 7 elements.

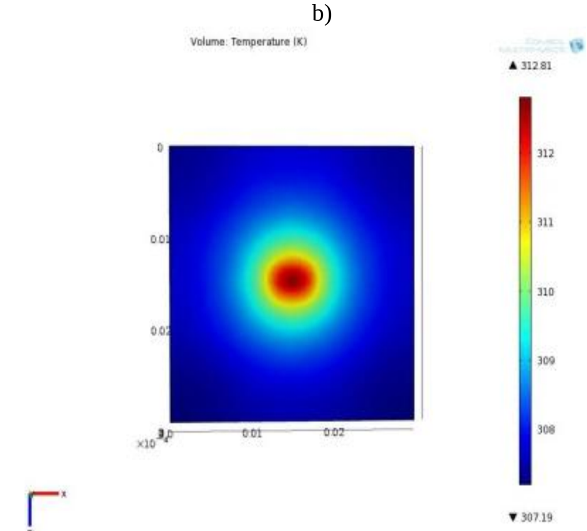
IV. RESULTS AND DISCUSSION

By the application of COMSOL Multiphysics is obtained the temperature distribution on the front side of the silicon sample excited by a modulated light beam with 0.05Hz of frequency. Fig. 2 shows the temperature distribution for three cases in the exposure time, 10s, 50s and 80s. Part of the incident light is absorbed in the surface of the sample and transformed in heat by means of nonradiative deexcitation processes. Because the incident light beam is modulated, the generated heat flux is periodic with 0.05Hz of frequency, which is propagated through the sample with 20*s* of period. Fig. 2(a) shows the temperature distribution on the front side of the silicon sample for 10s of the exposure time, that is, a half of period. The same for Fig. 2(b), at an exposure time of 50s (two and a half of period), and for Fig. 2(c), at an exposure time of 80s (four periods).

a)



b)



c)

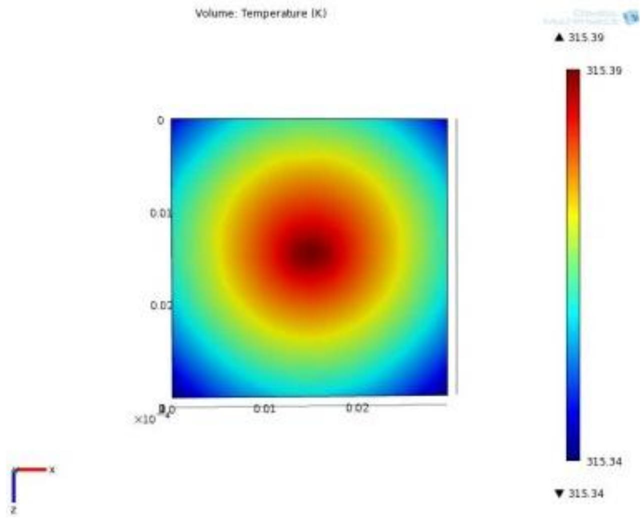
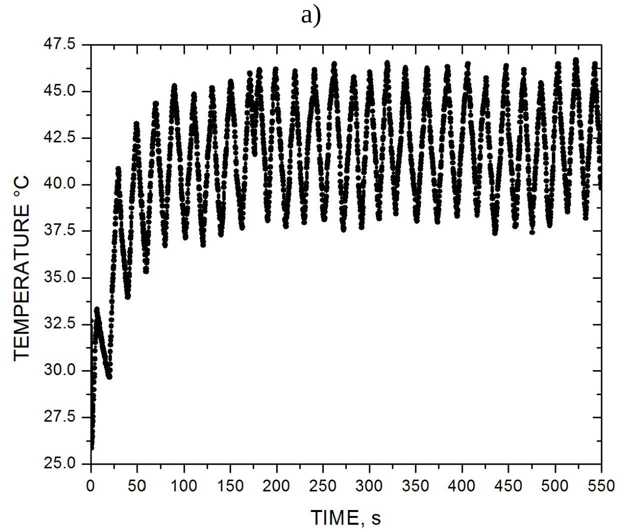


FIGURE 2. Temperature distribution on the front side of the sample at a modulation frequency of 0.05Hz. a) 10s b) 50s and c) 80s of exposure time.

Figure 3(a) shows the experimental results of the temperature evolution at the front face of the silicon sample as a function of the exposure time, obtained by mean of infrared Photothermal Radiometry using a modulated light beam with frequency of 0.05Hz. Fig. 3(b) shows the results obtained by COMSOL Multiphysics. From the comparison between these graphs it can be observed a similar behavior in the amplitude and in the shape of the curves too, which demonstrate a good agree between these results.

a)



b)

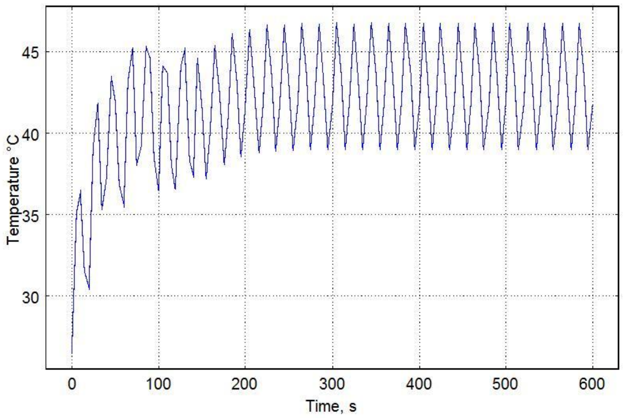


FIGURE 3. Temperature evolution at the front face of the sample in a time exposure of 550s and frequency of 0.05Hz. (a) Experimental and (b) Simulated result.

V. CONCLUSIONS

By the application of COMSOL Multiphysics software it was obtained the numerical solution of the periodic temperature distribution in a sample excited by a periodic light beam on its front surface with the congruent boundary conditions to the physical reality of the experimental setup. The good agree between the experimental and simulated results, shows the utility of the COMSOL Multiphysics software in the study of the thermal response in solids for measurements by means of Infrared Photothermal Radiometry.

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