Golden section heat engines and heat pumps

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Abstract

In this study the efficiency of heat engines and the coefficient of performance of heat pumps are reconsidered following the idea of the golden section ratio. Two models are analyzed: the golden section Carnot cycle and the golden section endo-reversible cycle. The observed expressions for efficiency and coefficient of performance are casted in terms of the golden section ratio. Single (steam turbines [40Wilfried, U., OMMI, Vol. 2, issue 3, pp. 1-9, 2003]) and double heat engine cycles (combined cycles [Ishikawa, M., Terauchi, M., Komori, T., Yasuraoka, J., Mitsubishi Heavy Industries, Ltd., Technical Review Vol. 45 No. 1, pp. 15-17, 2008]) are considered. Expressions for the working fluid temperatures at the hot side and the cold side are derived and it was found that the relative differences compared to the endo-reversible results falls in the range 2-4% only with relative power reduction of 15% from the maximum power point. Finally, it is shown that the system efficiency differs approximately by 3.6% from the efficiency derived based on the golden section.

Keywords: Heat engines, heat pumps.

Resumen

En este estudio, la eficiencia de los motores de calor y el coeficiente de rendimiento de las bombas de calor son consideradas siguiendo la idea de la proporción de la sección de oro. Dos modelos han sido analizados: El ciclo de Carnot la sección de oro y el ciclo de la sección de oro endo-reversible. Las expresiones observadas para la eficiencia y el coeficiente de rendimiento están difundidas en términos de la proporción de la sección de oro. Único (turbinas de vapor [40Wilfried, U., OMMI, Vol. 2, issue 3, pp. 1-9, 2003]) y dobles ciclos de motor térmico (ciclos combinados [Ishikawa, M., Terauchi, M., Komori, T., Yasuraoka, J., Mitsubishi Heavy Industries, Ltd., Revisión Técnica Vol. 45 No. 1, pp. 15-17, 2008]) se consideran. Expresiones para la temperaturas del fluido en el lado caliente y el lado frío se derivan y se encontró que las diferencias relativas comparadas con los resultados endo-reversibles cae en el rango 2-4% solo con reducción de potencia relativa de 15% desde la máxima power point. Finalmente, se muestra que la eficiencia del sistema difiere aproximadamente un 3.6% de la eficiencia derivada basada en la sección de oro.

Palabras clave: Máquinas de calor, bombas de calor.

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I. INTRODUCTION

The Carnot engine constitutes the basic heat engine model having reversible processes and yields upper bound of thermal efficiency for heat engines working between two heat reservoirs at constant temperatures. Practically a Carnot engine yields zero power. When the power output is limited by the heat transfer rates between the heat reservoirs and the working substance, the heat engine is defined as endo-reversible. Using the endo-reversible cyclic model, Novikov [1] and Curzon & Ahlborn [2] have obtained the efficiency $\eta_{C-A} = 1 - \sqrt{\frac{T_L}{T_H}}$ of a Carnot engine at maximum power output in which T_H and T_L are temperatures of heat source and sink, respectively. Following these studies, the methods of finite time thermodynamics (FTT) have been applied to a wide range of thermodynamic systems [3-27].

In a different approach, Popcov and Shipitsyn considered the Golden section in the Carnot cycle [28]. In their study, some aspects of classical thermodynamics are analyzed for the presence of duality and of the golden section.

Markowsky in his book review [35], gave some properties of the golden section. The number $(1 + \sqrt{5})/2 = 1.618\ldots$ is widely known as the *golden ratio*, ϕ and phi. Phi appears in many different equations and formulas and has many interesting properties. He claims that many people have heard marvelous tales about phi and how it permeates art and nature.

Hretcanu and Crasmareanu, [29] claims that the golden ratio (which is also known as the golden section) is a fascinating topic that continually generates new ideas. The main purpose of their paper was to point out and find some applications of the golden ratio and of Fibonacci numbers in differential geometry. They studied a structure defined on a class of Riemannian manifolds, called by them a golden structure. A Riemannian manifold endowed with a golden structure was called a golden Riemannian manifold.

Watson [30] states that the golden section is intimately related to growth and nature. More ever he claims that Eggs, an apple blossom, a human face, a seashell—all embody golden section proportions. He believes that the golden section is a simple tool that may be used to enhance the meaning and beauty of an architectural work. Based on that, he says that designing buildings with this knowledge automatically creates harmonizing, uplifting effects on those who experience them. More than that he claims that when the structural lines of a building are designed according to the principles of harmonic proportions, a natural aesthetic beauty results—beauty that can benefit those who work, live, and play within those environments. On the other hand, Kak S. [31], reported in his article that the golden mean, ϕ, has been applied in diverse situations in art, architecture and music, and although some have claimed that it represents a basic aesthetic proportion, others have argued that it is only one of a large number of such ratios.

Lefebvre and Efremov considered the problem of double geometric progression and golden section in the decay profiles of bursts [32]. They found that for bursts lasting less than 25sec and if two significant peaks are observed in the power spectrum, the ratio of their centroid frequencies is approximately constant and equal to 1.59. They found that this ratio is very close to the Golden Section ratio 1.618.

Smirnov [33] in his paper, reviews that Kepler analyzed planet movement parameters with music proportions. There exists no contradiction in such a method. It is known that the same different equations of oscillatory movement, for example, describe diverse physical phenomena. The mathematical basis of such a "musical" description of astronomical phenomena was the so-called "God's proportion" or "golden section". The golden section is found in bioengineering problems too. Bejan [34] claims that shapes with length/height ratios (L/H) close to 3/2 are everywhere and give the impression that they are being 'designed' to match the golden ratio (ϕ = 1.618). In his article he showed that these shapes emerge as part of an evolutionary phenomenon that facilitates the flow of information from the plane to the brain, in accordance with the constructal law of generation and evolution of design in nature.

The golden section is of interest to mathematicians and among them they are interested in finding new construction methods. Kaplan *et al*. [36] constructed the golden ratio by using a bisector area of a trapezoid. Hofstetter [37] gave a simple 5-step division of a segment into golden section, using ruler and rusty compass. In a different study [38] Hofstetter constructed, in 4 steps using ruler and compass, three points two of the distances between which bear the golden ratio. A different way of construction is given by Tong and Kung [39]. They construct the golden ratio by using an area bisector of a trapezoid.

In this study the efficiency of Carnot and endo-reversible heat engines and the coefficient of performance of Carnot heat pumps are cast with respect to the golden section ratio. The new expressions for efficiency are compared to the Curzon & Ahlborn efficiency. Although the observed efficiencies resulted from pure mathematical basis, it is interesting to note that the state of the art efficiency of steam turbines [40] and of the combined gas cycles [41] relatively differs by few percentages only.

The order of the article is as follows: In section 2 the golden section Carnot heat engine and heat is analyzed, in section 3 the golden section endo-reversible heat engine with Newtonian heat transfer is considered and a numerical example is given in section 4. Finally conclusion and discussion are given in section 4.

II. THE GOLDEN SECTION CARNOT HEAT ENGINE AND HEAT PUMP

Consider a Carnot heat engine that work between two heat reservoirs, a hot reservoir with temperature T_H and cold reservoir with temperature T_L. The schematic of the Carnot heat engine is given by Fig. 1. The heat input to the engine is Q_H and the heat output from the engine is Q_C. The work W output is calculated based on the energy equation and is given by:

$$W = Q_H - Q_C. \tag{1}$$

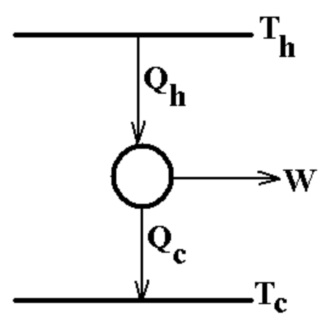

FIGURE 1. Schematics of the Carnot heat engine that working between two heat reservoirs, the hot reservoir at T_H and the cold reservoir at T_C.

The heat input to the engine is always divided into two parts. Due to losses of different types (finite heat transfer rates, friction, etc…) the amount of work output could be smaller or larger than the heat rejection. Without performing any massive calculations, one could follow the golden section division of a line and deduce expressions for the efficiency of the heat engine. The schematic of the golden section division of the heat input is depicted in Fig. 2. The golden section ratio for the case depicted in Fig. 2a is:

$$\frac{Q_H}{Q_C} = \frac{Q_C}{W}. \tag{2}$$

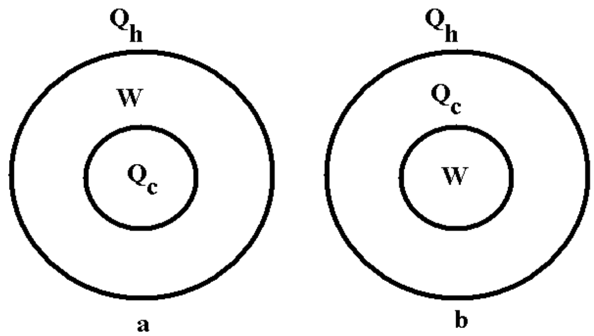

FIGURE 2. The golden section division if the heat input Q_H is done in two different ways: a. the heat rejection Q_C is larger than the work output W and b. the work output is larger than the heat rejection.

Equation 2 could be arranged in the following way after some manipulation:

$$\eta^2 - 3\eta + 1 = 0. \tag{3}$$

The physical solution of Eq. 2 gives the efficiency of golden section Carnot heat engine for the case that the heat rejection is larger than the work output. This efficiency is comparable with single cycle heat engines [40]. Performing the calculation leads to:

$$\eta_{single} = \frac{3-\sqrt{5}}{2} = 1 - \frac{1}{\phi} \approx 0.382. \tag{4}$$

Where η_{single} is the single cycle of golden section Carnot heat engines casted in terms of the golden section ratio, and ϕ is the golden section ratio (≈ 1.618).

For the Carnot heat engine there are six physical quantities: the amount of heat input to the engine, the amount of heat rejected from the engine, the work output, the reservoir's temperatures and efficiency. These quantities are related to each others with three equations: The energy conservation, the entropy conservation and the efficiency definition. For a given value of efficiency, heat input, the cold reservoir temperature and the two conservation equations, is it possible to solve for the other unknowns: the heat output, the work output and the hot reservoir temperature. After some mathematical manipulation, the hot reservoir temperature is given by:

$$T_H = \phi T_L. \tag{5}$$

If we reverse the arrows, we get a Carnot heat pump instead of a heat engine. The performance of a heat pump is usually given by the coefficient of performance. There are two coefficients of performance: one for heating and the other one for cooling. The coefficient of performance for heating COP_H is given by:

$$COP_H = \frac{Q_H}{W} = \frac{\phi}{\phi - 1}. \tag{6}$$

And the coefficient of performance for cooling COP_C is given:

$$COP_C = \frac{Q_C}{W} = \frac{1}{\phi - 1}. \tag{7}$$

For the other case, when the larger part of the heat input is turned to work output (as depicted in Fig. 2b), the golden section ratio is given by:

$$\frac{Q_H}{W} = \frac{W}{Q_C}. \tag{8}$$

Rearranging Eq. 8 leads to the following quadratic equation:

$$\eta^2 + \eta - 1 = 0. \tag{9}$$

The physical solution of Eq. 9 gives the efficiency of golden section Carnot heat engine for the case that the heat rejection is smaller than the work output. This efficiency is comparable with double cycle heat engines 41]. Performing the calculation leads to:

$$\eta_{double} = \frac{\sqrt{5}-1}{2} = \frac{1}{\phi} \approx 0.618. \tag{10}$$

Where η_{double} is the double cycle of golden section Carnot heat engines casted in terms of the golden section ratio.

For this case the temperature if the hot reservoir is given by:

$$T_H = \frac{\phi T_C}{\phi - 1}. \tag{11}$$

III. THE GOLDEN SECTION ENDO-REVERSIBLE HEAT ENGINE AND HEAT PUMP

In this section we consider an endo-reversible heat engine model. The model is described schematically by Fig. 3.

The endo-rersible heat engine runs between two heat reservoirs, hot reservoir at temperature T_H and cold reservoir at temperature T_L. The heat engine runs at finite rates, and the heat input and the heat rejection follow a Newtonian law. The heat input is given by:

$$Q_h = C_h\left(T_H - T_h\right). \tag{12}$$

Where C_h is the heat conductance at the hot side. Similarly, the heat rejection is given by:

$$Q_c = C_c\left(T_c - T_L\right). \tag{13}$$

Where C_c is the heat conductance at the cold side.

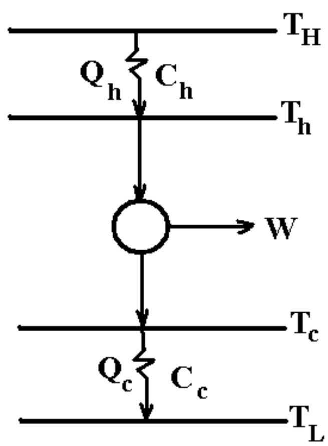

FIGURE 3. Schematic drawings for the endo-reversible heat engine. T_H The hot reservoir temperature, T_h is the working temperature at the hot side of the engine, T_C is the working temperature at the cold side of the heat engine, T_L is the temperature of the cold reservoir, C_h is the heat conductance at the hot side, C_c is the heat conductance at the cold side, Q_h is the heat input, Q_c is the heat rejection and W is the work output from the heat engine.

The work output is given by Eq. 1 which is calculated from the energy equation:

$$\Delta E = 0. \tag{14}$$

The entropy for the endo-reversible heat engine is conserved and is given by:

$$\Delta S = 0. \tag{15}$$

Equation 15 could be recast explicitly as follows:

$$\frac{Q_h}{Q_c} = \frac{T_h}{T_c} \tag{16}$$

Lastly, the efficiency of the endo-reversible heat engine is defined as the work output divided by the heat input and is given by:

$$\eta = \frac{W}{Q_h}. \tag{17}$$

Equations (12, 13, 14, 15, 16, 17) could be manipulated and the different physical amounts could be calculated explicitly from the following expressions.

The heat input is given by:

$$Q_h = \frac{C_h}{1 + C_h/C_c}\frac{(1-\eta)T_H - T_L}{1-\eta}. \tag{18}$$

The work output is given by:

$$W = \eta Q_h = \frac{C_h}{1 + C_h/C_c}\eta\frac{(1-\eta)T_H - T_L}{1-\eta}. \tag{19}$$

From Eqs. (18-19) it is obvious that for a given heat conductance at both sides hot and cold, the temperatures of the heat reservoirs and the efficiency of the heat engine; it is possible to calculate the heat input and the work output. Curzon & Ahlborn derived the efficiency at maximum power operation and it is given by (the Curzon & Ahlborn results are summarized for comparison purposes):

$$\eta_{C-A} = 1 - \sqrt{\frac{T_C}{T_H}}. \tag{20}$$

And the maximum power for the case $C_h = C_c$ is given by:

$$W = \frac{C_h}{2}T_H\left(1 - \sqrt{\frac{T_L}{T_H}}\right)^2. \tag{21}$$

The working temperature of the Curzon & Ahlborn heat engine at the hot side is given by:

$$T_h = \frac{T_H + \sqrt{T_H T_L}}{2}. \tag{22}$$

And the working temperature at the cold side is given by:

$$T_c = \frac{T_L + \sqrt{T_H T_L}}{2}. \tag{23}$$

In the following paragraphs the golden section endo-reversible heat engine is considered. Instead of seeking the efficiency at maximum power output, the efficiency is assumed to be given by Eq. 4. Then the work output of the golden section endo-reversible heat engine W_φ (for the case of equal heat conductance values at both sides of the engine) is given by:

$$W_\phi = \frac{C_h}{2} T_H (\phi - 1)\left(\frac{1}{\phi} - \frac{T_L}{T_H}\right), \tag{24}$$

The resulting working temperature at the hot side $T_{h\varphi}$ is given by:

$$T_{h\phi} = \frac{T_H + \phi T_L}{2}. \tag{25}$$

And the resulting working temperature at the cold side $T_{c\varphi}$ is given by:

$$T_{c\phi} = \frac{T_L + {T_H}/{\phi}}{2}. \tag{26}$$

Based on the equations given above, and for comparison purposes between the golden section endo-reversible heat engine and the endo-reversible heat engine, the following ratios are developed.

The work output ratio W_r is defined as the ratio between the work output of the golden section endo-reversible heat engine and the maximum work output of the endo-reversible engine is given by:

$$W_r = \frac{(\phi - 1)\left(\frac{1}{\phi} - \frac{T_L}{T_H}\right)}{\left(1 - \sqrt{\frac{T_L}{T_H}}\right)^2}, \tag{27}$$

The working temperature ratio at the hot side T_{hr} which is the ratio between the working temperature at the hot side of the golden section endo-reversible heat engine and the working temperature of the endo-reversible heat engine at the hot side is given by:

$$T_{hr} = \frac{1 + \varphi \frac{T_L}{T_H}}{1 + \sqrt{\frac{T_L}{T_H}}}. \tag{28}$$

The working temperature ratio at the cold side T_{cr} which is the ratio between the working temperature at the cold side of the golden section endo-reversible heat engine and the working temperature of the endo-reversible heat engine at the cold side is given by:

$$T_{cr} = \frac{1 + \frac{1}{\phi}\frac{T_H}{T_L}}{1 + \sqrt{\frac{T_H}{T_L}}}. \tag{29}$$

Finally, the efficiency of the system η_{sys} as defined in [12] is given by:

$$\eta_{sys} = \frac{\eta_C + \eta_{C-A}}{2}. \tag{30}$$

IV. NUMERICAL EXAMPLE

In this section we consider a numerical example for illustration and comparison purposes.

Consider a Carnot heat engine with hot reservoir at 600K and cold reservoir at 300K. For this case, the Carnot efficiency is 50%. The golden section efficiency (as given by Eq. 4 equals 38.2% and the Curzon & Ahlborn efficiency (as given by Eq. 20) equals 29.29% approximately. The system efficiency (calculated from Eq. 40) is 39.65% approximately. Thus the relative difference between the system efficiency and the golden section efficiency is approximately 3.6%.

For the golden section Carnot heat engine, the hot side reservoir temperature as estimated by Eqs. (5) and (11), equals 484.4K for the single cycle and 785.4 for the double cycle, respectively.

The endo-reversible heat engine predicts working temperature at the hot side as 710K and the working temperature at the cold side is predicted to be 410K. On the other hand, the results predicted from the golden section endo-reversible heat engines are 693K (Eq. 25) and 428K (Eq. 26) respectively. Thus the relative differences fall in the range of 2-4% approximately.

Finally, the work ratio (Eq. 39) predicts 85%. The golden section endo-reversible heat engine introduces only 15% reduction of the maximum work output.

V. CONCLUSION AND DISCUSSION

In this study a golden section Carnot heat engine and heat pump models were considered in section 2. Expression for the efficiency and the hot reservoir temperature were observed and vast as a function of the golden section ratio. For the Carnot heat engines two cases were considered: a single cycle and double cycle. The efficiency of the single cycle is observed to be 38.2%. The hot side temperature of the Carnot heat engine is predicted to be 484.4K based on 300K cold reservoir. For the double cycle case, the golden section efficiency of the Carnot cycle is 61.8% with 785.4K hot reservoir temperature.

The golden section endo-reversible heat engine model was considered in section 3. The observed efficiency of the heat engine working between 600K and 300K heat reservoirs is 38.2% which differs by about 3.6% relative to the system efficiency 39.65% (which is given by Eq. 40). The working fluid temperatures for the golden section endo-reversible heat engine and he endo-reversible engine were calculated and the relative difference between them falls in the range 2-4% at the price of 15% of the work output.

Similar analysis was performed for the Carnot heat pump and expressions for the coefficients of performance for heating and cooling were given by Eqs. (6) and (7).

Although the golden section ratio is a pure mathematical issue, it is interesting to note that the observed efficiencies of the single cycle and double cycle differs by small amount from the state of the art efficiencies of steam turbines and gas combined cycles. This can be seen from the studies of Wilfried [40] and Ishikawa *et al*., [41]. Wilfried [40] reports that the state of the art of steam turbines (single cycle) provided by steam parameters with supercritical pressures and temperatures up to 6100C reach higher cycle efficiencies. The improvement in efficiency is due to progress in material technology (for casing, rotors, and turbine blades) and computers (CFD simulations). Optimizing the turbine blades led to reduce inner losses with 96% turbine efficiency. Thus the system efficiency (net efficiency) could reach 43%. Ishikawa *et al*., [41] Gas turbine combined cycle power plants (double cycle) mainly using MHI D, F and G-type gas turbines are currently in operation in large numbers, and there are many under construction or planned both in Japan and abroad. They report that the efficiency of the combined cycle plant with a 1,700°C class gas turbine with its inlet temperature improved to 1,700°C compared with the F-type and G-type with inlet temperatures of 1,500°C has reached the level of 62% - 65% (LHV base), and is expected to stand at a higher level than the efficiency of the conventional combined cycle plant.

The golden section has been reported in the literature as a measure of beauty in different fields of science and nature. In this study the efficiency of heat engines evolved over the years to reach the level of golden section ratio. The big question here - Did heat engines reach the beauty level or the observed expressions for the efficiency are just fortuitous?!

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