Fibonacci's motion problem "Two travellers": The solutions given by junior high-school students who were trained for Mathematical Olympiad

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Abstract
Numerical problems are today an important part of physics textbooks. One of the most popular motion problems is related to the situations in which two motions are combined. Although numerical problems appeared in physics textbooks in the middle of XIX century, a particular problem situation (combination of a motion at a constant speed and a motion at a constant acceleration) was present in mathematics textbooks since the beginning of the XIII. In the design of the research instrument we used the problem formulation proposed by Fibonacci in 1203 (the problem of two travellers). In this article we report performances of junior high-school students who participated in the training program for Mathematical Olympiad at the Facultad de Ciencias Físico Matemáticas of the Benemérita Universidad Autónoma de Puebla (Puebla, México). This initial qualitative study presents the results of the analysis of strategies students used in solving the problem of two travellers. Beside various acceptable solutions (some conceptually clearer than the solution given by Fibonacci!), we analyzed also the errors made by students in order to know how they understood the problem and how planned to solve it.

Keywords Fibonacci, gifted students, problem solving.

Resumen
Problemas numéricos son hoy una parte importante de los libros de texto de física. Uno de los más populares problemas de movimiento está relacionado con la situación en que se combinan dos movimientos. Aunque problemas numéricos aparecieron en los libros de texto de física en la mitad del siglo XIX, una particular situación de problema (la combinación de un movimiento a velocidad constante con uno a aceleración constante) era presente en los libros de texto de matemática desde el principio del siglo XIII. En el diseño del instrumento de investigación hemos usado la formulación propuesta por Fibonacci en 1203 (el problema de dos viajeros). En este artículo reportamos el desempeño de alumnos de secundaria que participaban en el programa de entrenamientos para la olimpiada de matemáticas en la Facultad de Ciencias Físico Matemáticas. Este inicial estudio cualitativo presenta los resultados del análisis de las estrategias que utilizaron estudiantes al resolver el problema de dos viajeros. Aparte de soluciones aceptables (algunas ¡conceptualmente más claras que la solución dada por Fibonacci!), hemos analizados también los errores que estos estudiantes cometieron para conocer cómo ellos interpretaron el problema y cómo planeaban su solución.

Palabras claves Fibonacci, estudiantes talentosos, resolución de problemas.

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I. INTRODUCTION

Today numerical problems or end-of-chapter problems are an important part of physics textbooks, especially at the university level. It is generally believed that enough practice in solving them is the best way to get conceptual understanding of physics.

Basic mathematical structure and solving algorithm of many of these problems are repeated in textbooks, although their particular wordings and contextual make-ups may differ greatly to avoid plagiarism flavor or to show author's creativity in problem formulations.

Having so many numerical problems in today's physics textbooks, it is hard to imagine that they were introduced in these teaching and learning tools in the middle of the XIX century. This new textbook element appeared for the first time in the edition of the influential Ganot's textbook "*Traite elementaire du physique*", published in 1854 [1].

In the XIII edition, published in 1868, the number of problems, covering all domains of physics was, measured by today's standards, very modest: only 103 problems were included [2]. Almost all problems were direct applications of mathematical formulas expressing quantitatively physics concepts and experimental laws, without inclusion of any real-world context. If a context was mentioned, it was quite artificial, calculation-oriented or even absurd from the common point of view.

Here come only a few examples.

The Problem XXI [2, p. 907] is related to a conical champagne glass with diameter of 6 cm. It was supposed that mercury, water and oil were poured in the glass, having each one a depth of 5 cm. The task was to calculate the masses of all three liquids.

Another version of this situation is the problem XXXI [2 p. 909]. In a conical champagne glass with diameter of 8 cm and the inner depth of 12 cm, this time only mercury and water are poured in such a peculiar way that the mass of mercury resulted three times bigger than the mass of water. The task was to calculate the heights of mercury and water.

In the Problem IX [p. 903], the weights of a sphere made of platinum were given for being in the air and for being immersed in mercury. The task was to find the density of platinum.

It is important note that there was no single problem on motion included in the Ganot's collection.

The other French authors followed Ganot and very soon numerical problems became standard part of physics textbooks. For instance, in the elementary physics textbook, written by Boutan and D'Almeida and published in 1874 [3], there were 294 numerical problems. That number is more than twice bigger than the number of physics problems found in Ganot's textbook published only six years earlier.

Collections of physics problems gained popularity in USA, in order to complement existing textbooks with a few or none physics problems. In his "Examples in Physics" [4], Jones presented over thousand problems. Nevertheless, there was only one kinematics problem out of 183 mechanics examples.

A similar situation is noted in Gage's "1000 Exercises in Physics" [5], too. There were a few motion problems, mostly related to those caused by gravitation (free-fall and parabolic motion).

Problems that deal with the situations in that motions of two bodies are assumed, like one of which was used in the research whose results are reported in this article, appeared very rarely. Snyder and Palmer, in their collection "One Thousand Problems in Physics" [6], formulated only two problems planned for the motions of two bodies in free fall that were dropped to fall freely from the same point.

In one problem, students know difference between starting times and their task is to find the time for which the distance between the bodies would take a given value [6, Problem 40, p. 40]. In the other problem, the time in which the distance took a particular value was given and the difference between starting times was sought [6, Problem 41, p. 41].

Shearer, in a collection of 1,497 problems [7], also formulated only two problems that deal with two moving object (two trains). In one, the task is to calculate relative velocities of trains' performing one-dimensional motions in the same and opposite directions [7, Problem 61, p. 26], while in the other, relative trains' distance in two-dimensional motion is sought [7, Problem 62, p. 27].

II. TWO-MOTION PROBLEMS IN ACTUAL PHYSICS TEXTBOOKS

Although the problem and task designs, based on abstract formula-based "application" or related to absurd contexts started with Ganot's problems, are still present in contemporary physics textbooks, many authors try to formulate problems for those situations that might, in principle, happen in the real world.

Good examples of such type of problems are two-motion problems that were absent or rarely formulated in textbooks and problems collections at the end of XIX and the beginning of XX century.

A two-motion problem is the problem related to a situation in that two motions are present. It may be, for instance, a race of two persons or two cars. Involved motions, of course, may have different features (fixed or situation-dependent length of the race, start times, the ways the persons or cars move,…).

Two formulations of race problems are:

Constant-acceleration race, determined length and different starting positions

> "A Porsche challenges a Honda to a 400-m race. Because the Porsche's acceleration 3.5 m/s^2 is larger than the Honda's 3.0 m/s^2, the Honda gets a 50-m head start. Both cars start accelerating at the same instant. Who wins?" [8, Problem 70, p. 76]

Constant-speed race, undetermined length and different starting positions

> "Two cars, a Porsche and a Honda, are traveling in the direction, although the Porsche is 186 m behind the Honda. The speed of the Porsche is 24.4 m/s and the speed of the Honda is 18.6 m/s. How much time does it take for the Porsche to catch the Honda? [*Hint:* What must be true about the displacements of the two cars when they meet?]" [9, Problem 1, p. 95]

The most popular two-motion problems in physics textbooks are those related to well-known situation in that a police office is chasing a driver who broke the speed limit. In solved examples, it is usually supposed that (1) the speeder drives at a constant speed, and (2) the police officer starts from the rest and maintains a constant acceleration:

> "A speeder doing 40.0 mi/h (about 17.9 m/s) in a 25 mi/h zone approaches a parked police car. The instant the speeder passes the police car, the police begin their pursuit. If the speeder maintains a constant velocity, and the police car accelerates with a constant acceleration of 4.50 m/s^2, **(a)** how long does it take for the police car to catch the speeder, **(b)** how far have the two cars traveled in this time, and **(c)** what was the velocity of the police car when it catches the speeder?" [10, Example 2-9 **Catching a Speeder**, p. 36)

> "A motorist traveling at a constant velocity of 15 m/s passes a school-crossing corner where the speed limit is 10 m/s (about 22 mi/h). A police officer on a motorcycle stopped at the corner starts off in pursuit with constant

acceleration of 3.0 m/s^2… (**a**) How much time elapses before the officer catches up with the car? (**b**) What is the officer's speed at that point? (**c**) What is the total distance the officer has traveled at that point?" [11, Example 2.9 **Pursuit!**, p. 47]

In order to be original, it is common that textbook authors vary the relationship between given and sought data. Here come a few possible variants of the problem:

Constant speed and catching distance are given and constant acceleration is sought [12, Problem 31, p. 54]

Constant speed and constant acceleration are given and catching time, final speed and catching distance are sought [13, Problem 60, p. 55]

Constant acceleration and catching time are given and constant speed is sought [14, Problem 2.49, p. 59]

In all previous situations, police officers started from the rest. Nevertheless, there are textbook authors who think that chasing situation would be more realistic (or mathematically interesting), if the police car were in motion, either in the same direction [15, Problem 3.64, p. 42] or, even, in the opposite direction [16, Problem 62, p. 81]

Although one could cite more complicated (and, in real world less likely!) situations that are assumed by authors for two-motion problems, it is out of the scope and objectives of this article to go any further into discussion of the practices and learning implications of pursuit problems found in physics textbooks.

III. MOTION PROBLEMS IN FIBONACCI'S BOOK

Taking into account historic fact that the fist textbooks and collections of physics problems were almost without two-motion problems, it is surprising to learn that Fibonacci included a few two-motion problems in his, ground-breaking book "*Liber Abaci"*, in which Arabic numbers were introduced in European mathematics and whose first edition was published in 1203! In the Chapter XII of that book, whose first English translation appeared eight centuries later, in 2003 [17], it is possible to read these two-motion problems:

"*On Two Ships That Meet*
Two ships are some distance apart, which journey the first can complete in 5 days, the other in 7 days; it is sought in how many days they will meet if they begin the journey at the same day." [17, p. 280]

"*On Two Ants, One of Which Follows the Other*
Two ants are on the ground 100 paces apart, and they move in the same directions towards a single point; the first of them advances daily 1/3 of a pace and retreats 1/4; the other advances 1/5 and retreats 1/6; it is sought in how many days they will meet." [17, p. 280]

"*On Two Serpents*
… There is a serpent at the base of a tower that is 100 palms high, and he ascends daily 1/3 of a palm, and he descends daily 1/4. At the top of the tower there truly is another serpent who descends daily 1/5 of a palm, and ascends 1/6; it is sought in how many days they will meet in the tower." [17, p. 274]

The second and third problem shows clearly that fantastic contexts, impossible in real world but with supposed "appealing" mathematical properties, were introduced into and made common in mathematics teaching many centuries before they appeared in physics textbooks.

One of a few two-motion problems, used in this study, has the following formulation:

"*On Two Travellers, One of Whom Goes after the Other with and Increasing Pace*
… There are two men who propose to go on a long journey, and one will go 20 miles daily. The other truly goes 1 mile the first day, 2 second, 3 third, and so on always one more mile daily to the end when they meet; it is sought for how many days the first is followed…" [17, p. 261]

Fibonacci presented the solution of the problem by these considerations:
"(It) is found thus: namely, when the 20 is doubled there results 40 from which you subtract 1; there remains 39, and this amount of days he is followed; he who goes daily 20 miles goes in these days 39 days 20 times 39 miles, which make 780 miles. The other man truly in the same 39 days goes as many miles as are in the sum of the numbers which run from one up to 39, which sum is found similarly from the multiplication of the 20 by the 39." [17, p. 261].

Although numerically the solution given by Fibonacci is correct, it is not justified clearly why, in order to get it, one has to double the 20 days and then to subtract one day.

An additional version of the "two-traveller problem" and its solution are:

"… One goes daily 60 miles, and the other truly goes with an increase of three, that is in the first day 3 miles, in the second 6, in the third 9, and so forth,…"

"… You divide 60 by 3, there will be 20 that you double, there will be 40 from which you subtract one; there remains 39, and for this amount of days he will follow…" [17, p. 262]

Looking at the solution, it is possible to conclude that Fibonacci had the following idea:

To find the number n of days in which the paces became equal, one should divide the constant pace of the first traveller with the daily pace increment of the second traveller. The number $N = 2n - 1$ is then the number of days the second traveller needs to reach the first one.

Conceptually more transparent reason of this "correct" algorithm would be to say:

The distance deficit, accumulated in the first (n-1) days before the paces become equal in the n-th day, would be compensated if, after the n-th day, the second man travels (n-1) days more. So, the solution would be, again, (2n – 1) days, but it comes out from a meaningful summa [n days + (n – 1) days = (2 n – 1) days] and not from a conceptually opaque combination of a multiplication and a subtraction.

IV. THE SAMPLE OF STUDENTS AND THE DATA-COLLECTING INSTRUMENT

The data-collection instrument in this research was given to 44 junior high-school students, who participated in the training for mathematical Olympiad for that level at the Facultad de Ciencias Físico Matemáticas (Benemérita Universidad Autónoma de Puebla, Puebla, Mexico) in March of 2014. Students' ages were between 12 y 15 years.

All students accepted to participate and to do the tasks that form part of the instrument. They agreed that the parts of their individual works might be used anonymously as research data in conference contributions and scientific journal publications.

The structure of the instrument was partially inspired by the well know problem-solving steps proposed by George Polya [18]: 1. Understanding problem; 2. Devising a plan; 3. Carrying out the plan; 4. Looking back.

The idea behind was to explore what students would do if asked explicitly to follow these expert-like problem-solving steps. Namely, neither Mexican mathematics curriculum nor mathematics textbooks for junior high school promote them. The text of the data-collecting instrument is given in the **BOX 1**.

Two travellers
"There are two men who propose to go on a long journey, and one will go 20 miles daily. The other truly goes 1 mile the first day, 2 miles second, 3 miles third, and so on, always one more mile daily to the end when they meet. How many days does a second man need to reach the first one?"
a. Describe only in words (without using formulas or mathematical expressions) the plan you have to solve the problem.
b. Carry out the plan mathematically.
c. Your solution is: The second traveller reaches the first one after _____ days.
d. Show below that your solution is correct.

BOX 1. The text of the data-collecting instrument used in this research.

V. THE RESULTS

From 44 students, who presented their solutions, 12 solved the problem correctly, obtaining the solution of 39 days.

Some students used only "pure" solution strategies:
(1) Algebraic equation containing Gauss' formula;
(2) Making a table;
(3) Looking for sums that are multiple of 20;
(4) Looking for a simple pattern.

Others used "mixed" solution strategies, that were a combination of "pure" ones:
(5) Table and equation;
6) Table and "trial and error"; and
(7) Equation and "trial and error".

A. Correct solutions

A few examples of "pure" strategies are:
(1) Algebraic equation containing the formula of Gauss (Student 18)

a. The plan of this student is: "Firstly, the second traveller make an increment of one meter (instead of one mile!) and goes according the number of days, and the first advances the same distance. Finding the distances could be this way: 20 n for the first traveller and n(n+1)/2 for the second.

b. Mathematical execution of the plan:

$$20\,n = \frac{n(n+1)}{2}$$
$$40\,n = n(n+1)$$
$$40\,n = n^2 + n$$
$$39\,n = n^2$$
$$39 = \frac{n^2}{n}$$

c. The given solution is 39 days.
d. In the procedure to demonstrate that the solution is correct, the students wrote:

$$20\;x\;39 = 180 + 600 = 780 \qquad \frac{39\,(39+1)}{2} = \frac{1560}{2} = 780$$
$$780 = 780$$

Comment: This student, from the beginning, makes the plan to solve the problem using algebraic equation. It was a great pleasure for involved researchers to find out that junior high-school students carried out something what Fibonacci himself was not able to do because algebraic procedures were not common for him. Fibonacci knew how to find the sum of consecutive numbers when the last one is determined (the product of the last and the half of the sum of the last and the first) [17, pp. 259 - 261].

(2) Making a Table (Student 43)

a. The solution plan of this student is:
"(I will) make a table to compare the journey of both men."

b. Mathematical execution of the plan was the following table.

Day	Man 1	Man 2	Day	Man 1	Man 2
1	20	1	21	420	231
2	40	3	22	440	253
3	60	6	23	460	276
4	80	10	24	480	300
5	100	15	25	500	325
6	120	21	26	520	351
7	140	28	27	540	378
8	160	36	28	560	406
9	180	45	29	580	435
10	200	55	30	600	465
11	220	66	31	620	496
12	240	78	32	640	528
13	260	91	33	660	561
14	280	105	34	680	595
15	300	120	35	700	630
16	320	136	36	720	666
17	340	153	37	740	703
18	360	171	38	760	741
19	380	190	39	**780**	**780**
20	400	210			

c. The given solution was "39 days".
d. There is no verification of the result.
Comment: Elaborated table is totally complete. The operations were not given. So, it is not possible to know if the student knows or doesn't know the formula of Gauss.

(3) Looking for sums that are multiple of 20 (Student 19)
a. The plan "Firstly, I will find the sums of the consecutive numbers and will see if the result is a multiple of 20. After that I will count the days."
b. Although there are not verbal comments, it is possible to conclude that the student discovered a rule for calculating the distances covered in five consecutive days. Every next partial sum is bigger for 25:
1+2+3+4+5=15
6+7+8+9+10=40
11+12+13+14+15=65
16+17+18+19+20=90
21+22+23+24+25=115
26+27+28+29+30=140
31+32+33+34+35=165
36+37+38+39=150
The students does not justify why in the last sum only 4 days are taken.
In the next moves, the students find higher partial sums and the final sum:
15+40+65=120
120+90+115=210+115=325
325+140=465
465+165=630
630+150=780
The only two sums that satisfy the sought condition are 120 (for 15 days of traveling) and 780 (for 39 days of traveling). As 15 x 20 is 300, a number that is not equal to 120, the solution is 39 days (39 x 20 = 780).

c. The given solution is 39 days.
d. In order to demonstrate that the solution is correct, the students wrote:
39 x 20 = 780 The traveller who walks 20 miles daily.
$\frac{n(n+1)}{2} = \frac{39(40)}{2} = 780$ The traveller who makes one more mile daily.

Comment: This student uses the algebraic formula of Gauss only in the last part. At the beginning, the plan is to add the distances traveled by the second man and looking for those that are multiples of 20.

(4.1) Looking for a simple pattern (Student 37)
a. The plan: "*Write what the second travels and note that, as his pace increases, some day he will reach the first and note that what was the advantage (of the first) is equal what was the incremented."*
b. Carrying out mathematically the plan, the student writes:
"*Let us note that, in the first 20 days, the first traveler traveled* $20^2 = 400$ *and the second* (20)(21)/2 = 210. It means that the advantage was 190 *miles. In the next days, the second traveler travels more miles. Then, the quantity that made less in the first 19 days is equal to the miles that will make more in the next 19 days. Thus, the answer is*
(19 x 2) + 1 = 38 +1 = 39".
c. The solution is 39 days.
d. Demonstrating that the solution is correct, the student writes:
The first traveler travels 20 x 39 = 780 and the second 39 x 40 / 2 = 780.
Comment: Although the student knows the formula of Gauss, she or he is able to recognize the simple problem pattern that the accumulated advantage of the first traveler in the first 19 days will be compensate by summing the advantages the second traveler will make in 19 days after the 20th day. Such a clearly expressed conceptual insight into the problem pattern was missing in the solutions of Fibonacci.

(4.2) Looking for a simple pattern (Student 32)
a. Instead of a plan, the student states the basic feature of the problem: "At the beginning the first has an advantage, but later the second is going to walk more than before and will compensate that advantage.*"*
b. Representing that feature, the student wrote:

"Day	T_1	T_2
1	20	20-19
2	20	20-18
3	20	20-17
.		
19	20	20-1
20	20	20
21	20	20+1
22	20	20+2
.		
39	20	20+19."

c. The given solution is 39 days.
d. The demonstration, that the solution is correct, goes like this:

"T_1 = 39(20) = 780
T_2 = 39(20) +
+ (+19+18+17+16+15+14+13+12+11+10+9+8+7+6+5+4+3+2+1)
+ (-19 – 18-17-16 -15 -14 -13 -12 – 11- 10 –9-8-7- 6- 5 -4- 3-2-1) =
= 39(20) + (0) +…+ (0) = 780."

Comment: This student also got from the beginning the right insight into the symmetric problem pattern, something what Fibonacci could not get.

B. Most common errors revealed by the students

The most common failure in students' performances was misunderstanding of the problems announcement "The other truly goes 1 mile the first day, 2 mile second, 3 mile third, and so on…". They interpreted that the distances were note those walked daily, but those accumulated distance from the first day. In other words, these students interpreted that the second traveler always travels only one mile daily. Consequently, some of them erroneously concluded that the second traveler would never reach the first one.

The misunderstanding "one mile daily" goes, in some cases, with an additional misunderstanding.

(1) Two combined misunderstandings (Student 27)

a. Instead of a plan, a solution is given: "*If the second traveler walks daily 20 miles daily, then the other, if he walks one mile daily, has to walk 20 miles that would be 20 days. Namely, if he walks one mile daily, in order to reach the other, he must walk 20 days."*

b. The erroneous reasoning is repeated in the section where the plan had to be carried out. A table was presented.

Miles	Days
1	1
2	2
3	3
.	.
.	.
20	20

c. The given solution is **20** days.

d. Demonstration of the result was done verbally: If the second traveler covers one mile daily and the first 20 miles daily, he (the second traveler) needs 20 days to reach him (the first traveler).

Comment: It seems that this student has another misunderstanding: The first traveler walks 20 miles during the first day and then waits that the second traveler, covering one mile daily, reaches him after 19 more days.

Along with interpretative misunderstanding, some students also made calculation errors that were obstacles for finding the solution.

(2) Calculation errors (Student 10)

a. The plan reads: "I can do operations until finding the correct number. 20 x 20 = 200"

b. Execution of the plan were the following numerical patterns:

	1	2	3	4	5	6	7	8	9	10
	11	12	13	14	15	16	17	18	19	20
20 x 39 = 780	21	22	23	24	25	26	27	28	29	30
	31	32	33	34	35	36	37	38	39	

64 68 72 76 80 84 88 92 96

64+68+72+76 +80+84+88+92+96 = 0

c. No solution is given.

d. There is no verification.

Comment: In the plan there is a numerical error "20 x 20 = 200", very unexpected of someone interested in Mathematical Olympiad. There is no justification why the numbers in ten columns finish with the correct numbers of days (39). The student made correct sums of 9 number columns and didn't find the easiest one (10 + 20+30 =60).

VI. CONCLUSIONS

Although two-motion problems are very popular in today's physics textbooks, they started to appear there since the beginning of the XX century. It is not well known that this type of problems was used in mathematics textbooks, at least since the beginning of the XIII century, when they were included in Fibonacci's Liber Abaci.

Fibonacci provided correct solutions to a few versions of the problem "Two travellers", but these solutions did not have clear conceptual justifications.

In this research, the problem "Two travellers" was given to 44 junior high-school students who participated in a training program for Mathematical Olympiad at the Facultad de Ciencias Físico Matemáticas (Benemérita Universidad Autónoma de Puebla, Puebla, México).

The results show that some students were able to apply the Gauss formula and solved the problem through an algebraic approach. Other students were able to grasp a simple, symmetric feature of the problem situation that permits an easy arithmetic solution without using algebra. So, these students, in a sense, outperformed Fibonacci!

On the other side, some students revealed poor understanding of the problem situation and made calculation errors that are not expected from those who want to participate in Mathematical Olympiad.

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