The spherical derivation of the pressure expression $P = ⅓\ \rho C^2$ of a ideal gas

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Abstract

The Kinetic Molecular Theory (KMT) of gases is used to explain macroscopic properties of a gas, such as pressure and temperature. The existing derivations of the pressure expression are lengthy and based on imaginary components of velocities of ideal gas molecules. Due to these, the deduction of the said expression in lower study of physical science (*i.e.*, +2 syllabus) is avoided and introduced in pass degree course [1]. But an assumption of an ideal gas to be enclosed in an elastic sphere and collision of phenomena of gas molecules with its wall leads us very simply to the spherical derivation of the pressure expression $P = ⅓\ \rho\ C^2$ of an ideal gas from KMT.

Keywords: Kinetic Theory of Gases, Kinetic Molecular Theory, pressure of gases.

Resumen

La Teoría Cinética Molécular (TCG) de los gases se usa para explicar las propiedades macroscópicas de un gas, como la presión y la temperatura. Las derivaciones existentes de la expresión de presión son largas y se basan en componentes imaginarios de las velocidades de las moléculas de gas ideales. Debido a esto, la deducción de dicha expresión en el estudio inferior de la Ciencia Física (es decir, +2 programa) se evita e introduce en el curso de pregrado [1]. Pero la suposición de un gas ideal para ser encerrado en una esfera elástica y la colisión de los fenómenos de las moléculas de gas con su pared nos lleva muy sencillamente a la derivación esférica de la expresión de presión $P = ⅓\ \rho\ C^2$ de un gas ideal de TCG.

Palabras clave: Teoría cinética de los gases, teoría molecular cinética, presión de los gases.

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I. INTRODUCTION

The macroscopic property *pressure P*, of an ideal gas, is deduced from the postulates made in Kinetic Molecular Theory (KMT) [1, 2]. The phenomena of elastic collisions of gas molecules with wall of a sphere enclosed make change in its' momenta and hence pressure is exerted on the wall.

II. MATHEMATICAL MODEL

We follow the configuration of the molecules shown in figure 1 as a mathematical model, from where we can deduce the equations that govern the movement of molecules.

Let us assume that n molecules of an ideal gas are enclosed in an elastic sphere of radius r.

The centre of the sphere is O. Such is the case in figure 1. We draw C_1, C_2, C_3, …, C_n, as the velocity vectors of gas molecules about the centre O of the sphere. The gas molecule with velocity c1 makes a collision covering a diameter distance $2r$. The total momentum change for such an elastic collision is given by

$$mC_1 - m(-C_1) = 2mC_1.$$

Now the number of frequency of collisions of the molecule against the wall of the sphere per unit time is $C_1/2r$. Hence, the total momentum change for the said molecule per sec is given by $2mC_1\ C_1/2r = mC_1^2/r$.

Similarly that due to other molecule with velocity C_2 is mC_2^2/r and so on.

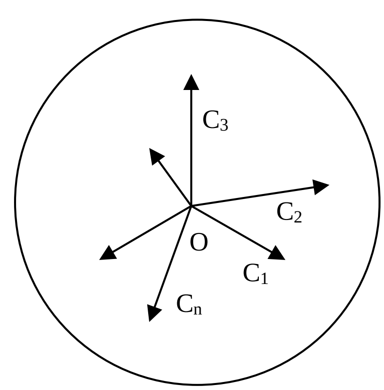

FIGURE 1. Kinetic arrangement of gas molecules.

Hence total momentum change per sec for *n* molecules, *i.e.* force on the wall of the sphere.

$$\begin{aligned} F &= mC_1^2/r + mC_2^2/r + mC_3^2/r + \cdots + mC_n^2/r, \\ &= mn/r\left\{\left(C_1^2 + C_2^2 + C_3^2 + \cdots + C_n^2\right)/n\right\}, \\ &= mn/r \cdot C^2. \end{aligned}$$

where the r.m.s. velocity C is given as

$$C = \left(\frac{C_1^2 + C_2^2 + C_3^2 + \cdots + C_n^2}{n}\right)^{1/2}.$$

Hence, the pressure on the wall, is given as

$$\begin{aligned} P &= \frac{Force}{Area}, \\ &= \frac{F}{4\pi r^2}, \\ &= \frac{\left(\frac{mnC^2}{r}\right)}{4\pi r^2}, \\ &= \frac{\frac{1}{3}mnC^2}{\frac{4}{3}\pi r^3}, \\ P &= \frac{\frac{1}{3}mnC^2}{V}, \end{aligned}$$

where V is the volume of the sphere and is given as

$$V = \frac{4}{3}\pi r^3.$$

Then if

$$\rho = \frac{mn}{V},$$

is the density of the gas. Then, the pressure is finally, given as follows

$$P = \frac{1}{3}\rho C^2.$$

III. CONCLUSIONS

This alternative derivation of the pressure expression is based on the consideration of actual velocities not on the imaginary components of velocity. In conventional derivations the constant 1/3 comes from the three components average. But here 1/3 comes directly from the geometric formula. This may be easier to understand for the students of even H. S. level (10+2 Class). It may have scope in microscopic world of physics.

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