Response to departures: a universal indicator

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SIC ITVR AD ASTRA
QVO NON ASCENDAM?

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Abstract

The fraction of energy that is not able to be transferred in an energy exchange between any two systems, be they of mechanical, electrical, optical, or quantum mechanical origin depends upon the perfect value of a specific ratio unique to each situation. This ratio can be of the mass of two colliding bodies on a straight line, the indices of refraction of two separating media, the external and the internal resistances in an electrical circuit, the energy of the incident particles and the energy of the one-dimensional potential barrier. And it is found that the ratio in each case has a perfect value of unity for which the fraction of energy that is not able to be transferred is zero and this loss increases with increase of departure from this unique value as revealed from the symmetry of a figure which is surprisingly identical for each case enumerated above.

Keywords: energy transfer, departure, perfect ratio, mechanical, electrical, optical, quantum mechanical.

Resumen

La fracción de energía que no se puede transferir en un intercambio de energía entre dos sistemas, ya sean de origen mecánico, eléctrico, óptico o mecanocuántico, depende del valor perfecto de una razón específica, única para cada situación. Esta razón puede ser la masa de dos cuerpos en colisión en línea recta, los índices de refracción de dos medios que se separan, las resistencias externa e interna de un circuito eléctrico, la energía de las partículas incidentes y la energía de la barrera de potencial unidimensional. Se ha descubierto que la razón en cada caso tiene un valor perfecto de la unidad, para el cual la fracción de energía que no se puede transferir es cero, y esta pérdida aumenta con la desviación de este valor único, como lo revela la simetría de una figura que es sorprendentemente idéntica para cada caso enumerado anteriormente.

Palabras clave: transferencia de energía, desviación, razón perfecta, mecánica, eléctrica, óptica, mecanocuántica.

I. INTRODUCTION:

The equations which appear in different fields of physics, and even in other sciences, are often almost exactly the same, so that many phenomena have analogues in these different fields. For example, oscillations of a mass on a spring, oscillations of charge flowing back and forth in an electrical circuit, vibrations of a tuning fork which is generating sound waves, the operation of a thermostat trying to adjust a temperature, vibrations of the electrons in an atom which generate light waves, growth of a colony of bacteria in interaction with the food supply and the poisons the bacteria produce, foxes eating rabbits eating grass follow equations which are very similar to one another [1]. In spite of seemingly diverse appearance all of them boast of an underlying symmetry linked to a linear differential equation with constant coefficients. Thus, it is important to realize that study of a phenomenon in one field permits an extension of one's knowledge in another field and justifies the reason for spending a great deal of time and energy on an understanding that is often taken for granted in the undergraduate teaching environment.

On the backdrop of this, a set of elegant illustrations involving transfer of energy from one system to another are chosen from mechanics, electricity, optics and quantum mechanics to show the underlying connection existing among them in spite of their diverse origin. We know that the energy exchange between any two systems is always fraught with possibility of leakage thereby lowering the efficiency of power delivered. Nevertheless, two systems participating in this transfer always have the scope to diminish this leakage to zero in order to attain one hundred percent efficiency [2]. Here, we have evaluated that unique scope for six different cases: two bodies of different masses undergoing one dimensional elastic collision, electromagnetic energy transmission between two optical media of different indices of refraction, incident particles of certain energy hitting one dimensional potential barrier, transfer of electrical energy from a cell of certain internal resistance to the external resistance in an electrical circuit; established the connection between leakage with departure from that unique scope for each case separately; and showed the surprising identical behaviour in seemingly different cases with each one leaving the same graphical footprint.

CASE 1: DEPARTURE FROM UNIT MASS RATIO

It is common knowledge that two bodies A and B of masses m_A and m_B and initial speeds v_{A1x} and v_{B1x} while undergoing one dimensional elastic collision always end with final speeds of v_{A2x} and v_{B2x} respectively given as [3]

$$v_{A2x} = \frac{m_A - m_B}{m_A + m_B} v_{A1x} + \frac{2m_B}{m_A + m_B} v_{B1x}$$
$$v_{B2x} = \frac{m_B - m_A}{m_A + m_B} v_{B1x} + \frac{2m_A}{m_A + m_B} v_{A1x}$$

Concentrating on the particular case in which body B is at rest before the collision ($v_{B1x} = 0$)or thinking body B as a target for body A to hit, the respective after-collision speed equations are

$$v_{A2x} = \frac{m_A - m_B}{m_A + m_B} v_{A1x}$$
$$v_{B2x} = \frac{2m_A}{m_A + m_B} v_{A1x}$$

So, the fraction of energy that is not able to be transferred to the target after collision is

$$\chi = \frac{\frac{1}{2}m_A v_{A1x}{}^2 - \frac{1}{2}m_B v_{B2x}{}^2}{\frac{1}{2}m_A v_{A1x}{}^2},$$

After simplification of mathematical steps this becomes

$$\chi = \left(\frac{1-n}{1+n}\right)^2,$$

where $n = \frac{m_B}{m_A}$, ratio of the target mass to the transferring mass. When$n = \frac{m_B}{m_A} = 1$,that is, the masses are equal, the transfer of energy is 100% efficient and the loss or uncoupled energy is zero.Thus, defining the perfect coupling as $n = n_o = 1$ and the departure from this perfect value as δn, we can write any value of the ratio n in terms of departure δn as$n = n_o + \delta n$.When $\delta n = 0$,$n = n_o = 1$. So, the loss equation in terms of departure can be written as

$$\chi = \frac{1}{\left(1 + \frac{2}{\delta n}\right)^2}$$

Where δn is the departure from the perfect value of the colliding mass ratio, $\frac{m_B}{m_A} = 1$

CASE 2: DEPARTURE FROM UNIT REFRACTIVE INDEX RATIO

Here we consider electromagnetic energy transmission when light is incident normally (for simplicity) in the z-direction on a medium having refractive index μ_1 and travels to another medium having refractive index μ_2. We consider both the medium non-magnetic. The incident, reflected and transmitted electric waves are given as [4]

$$\vec{E}_i(z,t) = E_i e^{i(k_1 z - \omega t)} \hat{x}$$
$$\vec{E}_r(z,t) = -E_r e^{i(-k_1 z - \omega t)} \hat{x}$$
$$\vec{E}_t(z,t) = E_t e^{i(k_2 z - \omega t)} \hat{x}$$

and since the magnetic field is related to the electric field as $\vec{B} = \frac{\vec{k} \times \vec{E}}{v}$ corresponding magnetic waves are given as

$$\vec{B}_i(z,t) = \frac{B_i}{v_1} e^{i(k_1 z - \omega t)} \hat{y}$$
$$\vec{B}_r(z,t) = -\frac{B_r}{v_1} e^{-i(k_1 z - \omega t)} \hat{y}$$
$$\vec{B}_t(z,t) = \frac{B_t}{v_2} e^{i(k_2 z - \omega t)} \hat{y}$$

Applying the boundary conditions at the boundary ($z = 0$) as $E_{\parallel 1} = E_{\parallel 2}$ and $B_{\parallel 1} = B_{\parallel 2}$ to the wave equations and solving for E_r and E_t and taking ratio of velocities in the two media as $\frac{v_1}{v_2} = \frac{\mu_2}{\mu_1}$, we get

$$E_r = \left(\frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}\right) E_i$$
$$E_t = \left(\frac{2\mu_1}{\mu_1 + \mu_2}\right) E_i$$

As we know, the average electromagnetic energy flux in the z-direction is given by

$$I = \frac{\vec{E} \times \vec{B}.\hat{z}}{\mu_0} = \frac{EB}{2\mu_0} = \frac{E^2}{2\mu_0 v}$$

So, we get coefficient of transmission T as

$$T = \frac{I_t}{I_i} = \frac{\mu_2}{\mu_1}\left(\frac{E_t}{E_i}\right)^2 = \frac{\mu_2}{\mu_1}\left(\frac{2\mu_1}{\mu_1 + \mu_2}\right)^2$$

So, the fraction of energy that is not able to be transferred is

$$\chi = 1 - T = 1 - \frac{\mu_2}{\mu_1}\left(\frac{2\mu_1}{\mu_1 + \mu_2}\right)^2 = 1 - \frac{4n}{(1+n)^2}$$
$$= \left(\frac{1-n}{1+n}\right)^2$$

Where $n = \frac{\mu_2}{\mu_1}$. And the loss equation in terms of departure can be written as

$$\chi = \frac{1}{\left(1 + \frac{2}{\delta n}\right)^2}$$

Where δn is the departure from the perfect value of the refractive index ratio, $\frac{\mu_2}{\mu_1} = 1$

CASE 3: DEPARTURE FROM UNIT ENERGY RATIO

Here we consider a quantum particle having mass m and momentum $\hbar k$ being incident from the left $x < 0$on the potential step defined as $V(x) = 0$ for $x < 0$ and $V(x) = V_o$ for > 0 . The Schrodinger equation satisfied by the particle is given as [5]

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}(E - V_o)\psi(x) = 0$$

The particle has kinetic energy $\frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m} = E - V_o$ and total energy E. As we know from elementary quantum mechanics the solution for the wave function for the particle in two regions defined by the potential are given as

$$\psi_I = e^{ikx} + re^{-ikx} \text{for} x < 0$$

$$\psi_{II} = te^{iqx} \text{for} x > 0$$

Where r and t are amplitudes of the reflected and transmitted waves respectively and $k = \frac{\sqrt{2mE}}{\hbar}$ and $q = \frac{\sqrt{2m(E-V_o)}}{\hbar}$. The unknown quantities r and t are determined by using the continuity of ψ and $\frac{d\psi}{dx}$ at $x = 0$. One get $r = \frac{k-q}{k+q}$ and $t = \frac{2k}{k+q}$.Since one can show that the incident current density is $j_{inc} = \frac{\hbar k}{m}$ and transmitted current density is $j_{tran} = \frac{\hbar q}{m}|t|^2$. The transmission coefficient T (which when multiplied with the incident energy is the fraction of energy transmitted) is given as

$$T = \frac{j_{tran}}{j_{inc}} = |t^2| = \frac{4kq}{(k+q)^2}$$

$$1 - T = 1 - \frac{4kq}{(k+q)^2} = 1 - \frac{4\frac{q}{k}}{\left(1+\frac{q}{k}\right)^2} = 1 - \frac{4n}{(1+n)^2}$$
$$= \left(\frac{1-n}{1+n}\right)^2$$

Where $n = \frac{q}{k}$. So, the fraction of energy that is not able to be transferred in terms of departure from perfect coupling can be written as

$$\chi = \frac{1}{\left(1+\frac{2}{\delta n}\right)^2}$$

Where δn is the departure from the perfect value of the ratio, $\frac{q}{k} = 1$

CASE 4: DEPARTURE FROM UNIT RESISTANCE RATIO

In an electrical circuit consisting of an emf $\mathcal{E}$ havimg an internal resistance r and an external resistance R in series, the current in the circuit is given as [6]

$$I = \frac{\mathcal{E}}{R+r}$$

The power transferred to the external resistance R is given as

$$P_{trans} = I^2 R = \frac{\mathcal{E}^2}{(R+r)^2} R = \frac{\mathcal{E}^2\left(\frac{R}{r}\right)}{r\left(1+\frac{R}{r}\right)^2}$$

So, the energy that is not able to be transferred to the external resistance is

$$\chi = 1 - \frac{P_{trans}t}{P_{available}t} = 1 - \frac{\frac{\mathcal{E}^2\left(\frac{R}{r}\right)}{r\left(1+\frac{R}{r}\right)^2}t}{\frac{\mathcal{E}^2 t}{r}} = 1 - \frac{\left(\frac{R}{r}\right)}{\left(1+\frac{R}{r}\right)^2}$$

$$= \frac{\left(1+\frac{R}{r}\right)^2 - 4\frac{R}{r} + 3\frac{R}{r}}{\left(1+\frac{R}{r}\right)^2} = \frac{\left(1-\frac{R}{r}\right)^2}{\left(1+\frac{R}{r}\right)^2} + \frac{3\frac{R}{r}}{\left(1+\frac{R}{r}\right)^2}$$

Considering the second term

$$\frac{3\frac{R}{r}}{\left(1+\frac{R}{r}\right)^2} = \frac{3\frac{R}{r}}{\frac{R^2}{r^2}\left(1+\frac{r}{R}\right)^2} = \frac{3\frac{r}{R}}{\left(1+\frac{r}{R}\right)^2}$$

Now generally in practical case the internal resistance of a battery is very small compared to the external resistance. So $\frac{r}{R} \to 0$ and we get

$$\chi = \frac{\left(1-\frac{R}{r}\right)^2}{\left(1+\frac{R}{r}\right)^2} = \left(\frac{1-n}{1+n}\right)^2$$

where $n = \frac{R}{r}$. So, the departure equation is written as

$$\chi = \frac{1}{\left(1+\frac{2}{\delta n}\right)^2}$$

Where δn is the departure from the perfect value of the resistance ratio, $\frac{R}{r} = 1$.

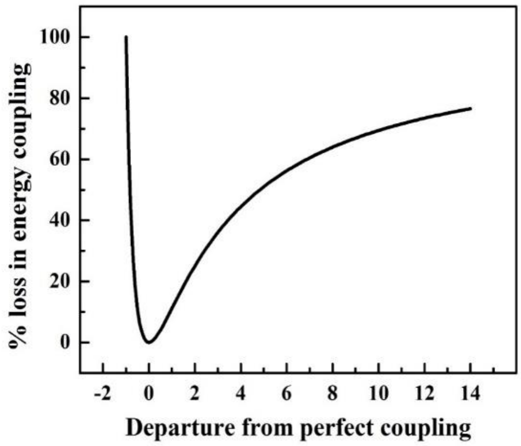

FIGURE 1. Case 1: Response of uncoupled power to departure from perfect matching in one dimensional elastic collision. At perfect matching condition (n=m_B/m_A =1, i.e.,δn=0) the uncoupled power is zero for two identical masses undergoing a one-dimensional elastic collision
Case 2: Response of uncoupled power to departure from perfect refractive indices matching. At perfect matching condition (n=μ_2/μ_1 =1, i.e.,δn=0) the uncoupled power is zero as there is now a single medium.
Case 3: Response of uncoupled power to departure from perfect matching. At perfect matching condition(n=q/k=1,δn=0) the uncoupled power is zero as there is no potential barrier.

Case 4: Response of uncoupled power to departure from perfect matching. At perfect matching condition (n=R/r=1,δn=0) the uncoupled power is zero

CONCLUSION

The case of two masses undergoing a one-dimensional elastic collision has shown that the power transfer always prefers equality of masses and abhors departures by lowering efficiency. Similarly, the conclusions of placing two media of different refractive indices never achieve a perfect coupling and the maximum transfer of electrical energy occurs when the external resistance equals the internal resistance and the more a potential barrier departs from the energy of the incident particles more is the energy that is not able to be coupled can be invoked to demonstrate the fact that a principle is not restricted to a small part of physics rather offering scope of an extension of knowledge from one field to another always. It also raises a question that this unique response to departures is merely coincidental or an offshoot of a fundamental insight?

REFERENCES

1.

[1] R. P. Feynman, R. B. Leighton, and M. Sands, The Feynman Lectures on Physics, Vol. I, Addison-Wesley Publishing Company, California, 1975.

[2] Subodha Mishra, Significance of a Recurring Function in Energy Transfer, The Physics Teacher 55, 298 (2017).

[3] Young HD, Freedman RA, and Ford Al, Sears and Zemansky's University Physics with Modern Physics, 13th Edison, Pearson.

[4] Born Max & Wolf Emil, Principles of Optics, 7th expanded edition, Cambridge University Press, 1997.

[5] Ghatak, Ajoy, Basic Quantum mechanics, Macmillan Publishers India Ltd, Paperback edition, New Delhi, 2010.

[6] Griffiths, David J, Introduction to Electrodynamics, 3rd edition, PHI Learning Pvt. Ltd., New Delhi, 2012.